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A supersymmetric extension of the standard model with bilinear R-parity violation

Chao-hsi Chang[1,2], Tai-fu Feng[2*]

[1] *CCAST (World Laboratory), P.O.Box 8730, Beijing 100080, P. R. China*

[2] *Institute of Theoretical Physics, Academia Sinica, P.O.Box 2735, Beijing 100080, P. R. China*

Abstract

The minimum supersymmetric standard model with bilinear R-parity violation is studied systematically. Considering low-energy supersymmetry, we examine the structure of the bilinear R-parity violating model carefully. We analyze the mixing such as Higgs bosons with sleptons, neutralinos with neutrinos and charginos with charged leptons in the model. Possible and some important physics results such as the lightest Higgs may heavy than the weak Z-boson at tree level etc are obtained. The Feynman rules for the model are derived in $'$t Hooft- Feynman gauge, which is convenient if perturbative calculations are needed beyond the tree level.

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*the contact author: fengtf@itp.ac.cn

I. INTRODUCTION

It is being increasingly realized by those engaged in the search for supersymmetry (SUSY) [1] that the principle of R-parity conservation, assumed to be sacrosanct in the prevalent search strategies, is not inviolable in practice. The R-parity of a particle is defined as $R = (-1)^{2S+3B+L}$ [2] and can be violated if either baryon (B) or lepton (L) number is not conserved. In recent years, the intensive studies of the supersymmetry that charactered by the bilinear R-parity violating terms in the superpotential and the nonzero vacuum expectation values (VEVs) of sneutrinos [3] have been undertaken. It stands as a simple supersymmetric (SUSY) model without R-parity which contains all particles as those in the standard model, and can be arranged in a way that there is no contradiction with the existing experimental data [4]. An impact of the R-parity violation on the low energy phenomenology is twofold in the model. One leads the lepton number violation (LNV) explicitly. The other is that the bilinear R-parity violation terms in the superpotential and soft breaking terms generate nonzero vacuum expectation values for the sneutrino fields $<\tilde{\nu}_i> \neq 0$ $(i = e, \mu, \tau)$ and cause the new type mixing, such as neutrinos with neutralinos, charged leptons with charginos and sleptons with Higgs etc.

The R-conserving superpotential for the minimal supersymmetric standard model (MSSM) has the following form in superfields:

$$\begin{aligned}\mathcal{W}_{MSSM} = \mu\varepsilon_{ij}\hat{H}_i^1\hat{H}_j^2 + l_I\varepsilon_{ij}\hat{H}_i^1\hat{L}_j^I\hat{R}^I - u_I(\hat{H}_1^2C^{JI*}\hat{Q}_2^J - \hat{H}_2^2\hat{Q}_1^I)\hat{U}^I \\ -d_I(\hat{H}_1^1\hat{Q}_2^I - \hat{H}_2^1C^{IJ}\hat{Q}_1^J)\hat{D}^I. \qquad (1)\end{aligned}$$

Where $\hat{H}^1$, $\hat{H}^2$ are Higgs superfields; $\hat{Q}^I$ and $\hat{L}^I$ are quark and lepton superfields respectively (I=1, 2, 3 is the index of generation), and all of them are in SU(2) weak-doublet. The rest superfields whereas $\hat{U}^I$ and $\hat{D}^I$ for quarks and $\hat{R}^I$ for charged leptons are in SU(2) weak-singlet. Here the indices i, j are contracted in a general way for SU(2) group and C^{IJ} $(I, J = 1, 2, 3)$ are the elements of the CKM matrix. However, when R-breaking interactions are considered, the superpotential is modified as the follows [5]:

$$\mathcal{W} = \mathcal{W}_{MSSM} + \mathcal{W}_L + \mathcal{W}_B \tag{2}$$

with

$$\begin{aligned} \mathcal{W}_L &= \varepsilon_{ij}[\lambda_{IJK}\hat{L}_i^I\hat{L}_j^J\hat{R}^K + \lambda'_{IJK}\hat{L}_i^I\hat{Q}_j^J\hat{D}^K + \epsilon_I\hat{H}_i^2\hat{L}_j^I] \\ \mathcal{W}_B &= \lambda''_{IJK}\hat{U}^I\hat{D}^J\hat{D}^K. \end{aligned} \tag{3}$$

Since the proton decay experiments set down a very stringent limit on the byron number violation [25], we suppress the term $\mathcal{W}_B$ totally. The first two terms in $\mathcal{W}_L$ have received a lot of attention recently, and restrictions have been derived on them from existing experimental data [6]. However, the term $\epsilon_I\varepsilon_{ij}\hat{H}_i^2\hat{L}_j^J$ is also a viable agent for R-parity breaking. It is particularly interesting because it can result in observable effects that are not to seen with the trilinear terms alone. One of these distinctive effects is that, the lightest neutralino can decay invisibly into three neutrinos at the tree level, which is not possible if only the trilinear terms in $\mathcal{W}_L$ are presented. The significance of such bilinear R-parity violating interaction is further emphasized by the following observations:

- Although it may seem possible to rotate away the $\hat{H}_2\hat{L}$ terms by redefining the lepton and Higgs superfields [7], their effect is bound to show up via the soft breaking terms.
- Even if one may rotate these terms away at one energy scale, they will reappear at another one as the couplings evolving radiatively [8].
- The bilinear terms give rise to the trilinear terms at the one-loop level [9].
- It has been argued that if one wants to subsume R-parity violation in a grand unified theory (GUT), then the trilinear R-parity violating terms come out to be rather small in magnitude ($\sim 10^{-3}$ or so) [10]. However, the superrenormalizable bilinear terms are not subjected to such requirements.

In this paper, we will keep $\epsilon_I\varepsilon_{ij}\hat{L}_i^I H_j^2$ as the only R-parity violating terms to study the phenomenology of the model. The plan of this paper is follows. In Sect.II, we will describe

the basic ingredients of the supersymmetry with bilinear R-parity violation. The mass matrices of the CP-even, CP-odd and charged Higgs are derived. Some interesting relations for CP-even and CP-odd Higgs masses are obtained. For completeness, we also give the mixing matrices of charginos with charged leptons and neutralinos with neutrinos. In Sect.III, we will give the Feynman rules for the interaction of the Higgs bosons (sleptons) with the gauge bosons, and the charginos, neutralinos with gauge bosons or Higgs bosons (sleptons). The self interactions of the Higgs and the interactions of chargino (neutralino)-squark-quark are also given. In Sect.IV, we will analyze the particle spectrum by the numerical method under a few assumptions about the parameters in the model. We find that the possibility with large value for ϵ_3 and $\upsilon_{\tilde{\nu}_\tau}$ still survives under strong experimental restrictions for the masses of τ-neutrino: $m_{\nu_\tau} \leq 20$ MeV and τ-lepton: $m_\tau = 1.77$ GeV. Finally we will close our discussions with comments on the model.

II. THE SUSY WITH BILINEAR R-PARITY VIOLATION

As stated above, we are to consider a superpotential of the form:

$$\begin{aligned}\mathcal{W} = \mu\varepsilon_{ij}\hat{H}_i^1\hat{H}_j^2 + l_I\varepsilon_{ij}\hat{H}_i^1\hat{L}_j^I\hat{R}^I - u^I(\hat{H}_1^2 C^{JI*}\hat{Q}_2^J - \hat{H}_2^2\hat{Q}_1^I)\hat{U}^I \\ -d^I(\hat{H}_1^1\hat{Q}_2^I - \hat{H}_2^1 C^{IJ}\hat{Q}_1^J)\hat{D}^I + \epsilon_I\varepsilon_{ij}\hat{H}_i^2\hat{L}_j^I \end{aligned} \tag{4}$$

with μ, ϵ_I are the parameters with units of mass, u^I, d^I and l^I are the Yukawa couplings as in the MSSM with R-parity. In order to break the supersymmetry, we introduce the soft SUSY-breaking terms:

$$\begin{aligned}\mathcal{L}_{soft} = & -m_{H^1}^2 H_i^{1*}H_i^1 - m_{H^2}^2 H_i^{2*}H_i^2 - m_{L^I}^2\tilde{L}_i^{I*}\tilde{L}_i^I - m_{R^I}^2\tilde{R}^{I*}\tilde{R}^I \\ & -m_{Q^I}^2\tilde{Q}_i^{I*}\tilde{Q}_i^I - m_{D^I}^2\tilde{D}^{I*}\tilde{D}^I - m_{U^I}^2\tilde{U}^{I*}\tilde{U}^I + (m_1\lambda_B\lambda_B \\ & +m_2\lambda_A^i\lambda_A^i + m_3\lambda_G^a\lambda_G^a + h.c.) + \{B\mu\varepsilon_{ij}H_i^1H_j^2 + B_I\epsilon_I\varepsilon_{ij}H_i^2\tilde{L}_j^I \\ & +\varepsilon_{ij}l_{sI}\mu H_i^1\tilde{L}_j^I\tilde{R}^I + d_{sI}\mu(-H_1^1\tilde{Q}_2^I + C^{IK}H_2^1\tilde{Q}_1^K)\tilde{D}^I \\ & +u_{sI}\mu(-C^{KI*}H_1^2\tilde{Q}_2^I + H_2^2\tilde{Q}_1^I)\tilde{U}^I + h.c.\}\end{aligned} \tag{5}$$

where $m^2_{H^1}, m^2_{H^2}, m^2_{L^I}, m^2_{R^I}, m^2_{Q^I}, m^2_{D^I}$, and $m^2_{U^I}$ are the parameters with units of mass squared while m_3, m_2, m_1 denote the masses of λ^a_G, λ^i_A and λ_B, the $SU(3)\times SU(2)\times U(1)$ gauginos. B and B_I are free parameters with units as mass. d_{sI}, u_{sI}, l_{sI} $(I = 1, 2, 3)$ are the soft breaking parameters that give the mass splitting between the quarks, leptons and their supersymmetric partners. The rest parts (such as the part of gauge, matter and the gauge-matter interactions etc) in the model are the same as the MSSM with R-parity and we will not repeat them here.

Thus the scalar potential of the model can be written as

$$\begin{aligned} V &= \sum_i |\frac{\partial\mathcal{W}}{\partial A_i}|^2 + V_D + V_{soft} \\ &= V_F + V_D + V_{soft} \end{aligned} \tag{6}$$

where A_i denote the scalar fields, V_D is the usual D-terms, V_{soft} is the SUSY soft breaking terms given in Eq. (5). Using the superpotential Eq. (4) and the soft breaking terms Eq. (5), we can write down the scalar potential precisely.

The electroweak symmetry is broken spontaneously when the two Higgs doublets H^1, H^2 and the sleptons acquire nonzero vacuum expectation values (VEVs):

$$H^1 = \begin{pmatrix} \frac{1}{\sqrt{2}}(\chi^0_1 + \upsilon_1 + i\phi^0_1) \\ H^1_2 \end{pmatrix} \tag{7}$$

$$H^2 = \begin{pmatrix} H^2_1 \\ \frac{1}{\sqrt{2}}(\chi^0_2 + \upsilon_2 + i\phi^0_2) \end{pmatrix} \tag{8}$$

and

$$\tilde{L}^I = \begin{pmatrix} \frac{1}{\sqrt{2}}(\chi^0_{\tilde{\nu}_I} + \upsilon_{\tilde{\nu}_I} + i\phi^0_{\tilde{\nu}_I}) \\ \tilde{L}^I_2 \end{pmatrix} \tag{9}$$

where $\tilde{L}^I$ denote the slepton doublets and $I = e$, μ, τ, the generation indices of the leptons. From Eq. (5), Eq. (6), we can find the scalar potential includes the linear terms as following:

$$V_{tadpole} = t^0_1\chi^0_1 + t^0_2\chi^0_2 + t^0_{\tilde{\nu}_e}\chi^0_{\tilde{\nu}_e} + t^0_{\tilde{\nu}_\mu}\chi^0_{\tilde{\nu}_\mu} + t^0_{\tilde{\nu}_\tau}\chi^0_{\tilde{\nu}_\tau} \tag{10}$$

where

$$
\begin{aligned}
t_1^0 &= \frac{1}{8}(g^2+g'^2)\upsilon_1(\upsilon_1^2-\upsilon_2^2+\sum_I \upsilon_{\tilde{\nu}_I}^2)+|\mu|^2\upsilon_1+m_{H^1}^2\upsilon_1-B\mu\upsilon_2-\sum_I \mu\epsilon_I\upsilon_I,\\
t_2^0 &= -\frac{1}{8}(g^2+g'^2)\upsilon_2(\upsilon_1^2-\upsilon_2^2+\sum_I \upsilon_{\tilde{\nu}_I}^2)+|\mu|^2\upsilon_2+m_{H^2}^2\upsilon_2-B\mu\upsilon_1+\sum_I \epsilon_I^2\upsilon_2\\
&\quad +\sum_I B_I\epsilon_I\upsilon_I,\\
t_{\tilde{\nu}_I}^0 &= \frac{1}{8}(g^2+g'^2)\upsilon_{\tilde{\nu}_I}(\upsilon_1^2-\upsilon_2^2+\sum_I \upsilon_{\tilde{\nu}_I}^2)+m_{L^I}^2\upsilon_{\tilde{\nu}_I}+\epsilon_I\sum_J \epsilon_J\upsilon_{\tilde{\nu}_J}\\
&\quad -\mu\epsilon_I\upsilon_1+B_I\epsilon_I\upsilon_2. \qquad (11)
\end{aligned}
$$

Here t_i^0 $(i=1,\,2,\,\tilde{\nu}_e,\,\tilde{\nu}_\mu,\,\tilde{\nu}_\tau)$ are tadpoles at the tree level and, the VEVs of the neutral scalar fields should satisfy the conditions $t_i^0=0$ $(i=1,\,2,\,\tilde{\nu}_e,\,\tilde{\nu}_\mu,\,\tilde{\nu}_\tau)$, therefore one can obtain:

$$
\begin{aligned}
m_{H^1}^2 &= -(|\mu|^2-\sum_I \epsilon_I\mu\frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_1}-B\mu\frac{\upsilon_2}{\upsilon_1}+\frac{1}{8}(g^2+g'^2)(\upsilon_1^2-\upsilon_2^2+\sum_I \upsilon_{\tilde{\nu}_I}^2)),\\
m_{H^2}^2 &= -(|\mu|^2+\sum_I \epsilon_I^2+\sum_I B_I\epsilon_I\frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_2}-B\mu\frac{\upsilon_1}{\upsilon_2}-\frac{1}{8}(g^2+g'^2)(\upsilon_1^2-\upsilon_2^2+\sum_I \upsilon_{\tilde{\nu}_I}^2)),\\
m_{L^I}^2 &= -(\frac{1}{8}(g^2+g'^2)(\upsilon_1^2-\upsilon_2^2-\sum_I \upsilon_{\tilde{\nu}_I}^2)+\epsilon_I\sum_J \epsilon_J\frac{\upsilon_{\tilde{\nu}_J}}{\upsilon_{\tilde{\nu}_I}}-\epsilon_I\mu\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_I}}\\
&\quad +B_I\epsilon_I\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_I}}). \qquad\qquad (I=e,\mu,\tau)
\end{aligned}
\tag{12}
$$

For convenience, we will call all of these scalar bosons (H^1, H^2 and $\tilde{L}^I$) as Higgs below. Now, we will give the Higgs boson mass matrix explicitly. For the scalar sector, the mass squared matrices may be obtained by:

$$
\mathcal{M}_{ij}^2=\frac{\partial^2 V}{\partial\phi_i\phi_j}|_{minimum}, \tag{13}
$$

here "minimum" means to evaluate the values at $<H_1^1>=\frac{\upsilon_1}{\sqrt{2}}$, $<H_2^2>=\frac{\upsilon_2}{\sqrt{2}}$, $<\tilde{L}_1^I>=\frac{\upsilon_{\tilde{\nu}_I}}{\sqrt{2}}$ and $<A_i>=0$ (A_i represent for all other scalar fields). Thus the squared mass matrices of the CP-even and the CP-odd scalar bosons both are 5×5, whereas matrix of the charged Higgs is 8×8.

A. The squared mass matrices of Higgs

From the scalar potential Eq. (6), we can find the mass terms:

$$\mathcal{L}_m^{even} = -\Phi_{even}^{\dagger}\mathcal{M}_{even}^2\Phi_{even} \tag{14}$$

where "current" CP-even Higgs fields $\Phi_{even}^T = (\chi_1^0,\ \chi_2^0,\ \chi_{\tilde{\nu}_e}^0,\ \chi_{\tilde{\nu}_\mu}^0,\ \chi_{\tilde{\nu}_\tau}^0)$. The mass matrix in Eq. (14) is

$$\mathcal{M}_{even}^2 = \begin{pmatrix} r_{11} & -e_{12} - B\mu & e_{13} - \mu\epsilon_1 & e_{14} - \mu\epsilon_2 & e_{15} - \mu\epsilon_3 \\ -e_{12} - B\mu & r_{22} & -e_{23} + B_1\epsilon_1 & -e_{24} + B_2\epsilon_2 & -e_{25} + B_3\epsilon_3 \\ e_{13} - \mu\epsilon_1 & -e_{23} + B_1\epsilon_1 & r_{33} & e_{34} + \epsilon_1\epsilon_2 & e_{35} + \epsilon_1\epsilon_3 \\ e_{14} - \mu\epsilon_2 & -e_{24} + B_2\epsilon_2 & e_{34} + \epsilon_1\epsilon_2 & r_{44} & e_{45} + \epsilon_2\epsilon_3 \\ e_{15} - \mu\epsilon_3 & -e_{25} + B_3\epsilon_3 & e_{35} + \epsilon_1\epsilon_3 & e_{45} + \epsilon_2\epsilon_3 & r_{55} \end{pmatrix} \tag{15}$$

with notations

$$\begin{aligned}
r_{11} &= \frac{g^2 + g'^2}{8}(3\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2) + |\mu|^2 + m_{H^1}^2 \\
&= \frac{g^2 + g'^2}{4}\upsilon_1^2 + \sum_I \mu\epsilon_I\frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_1} + B\mu\frac{\upsilon_2}{\upsilon_1}, \\
r_{22} &= \frac{g^2 + g'^2}{8}(-\upsilon_1^2 + 3\upsilon_2^2 - \sum_I \upsilon_{\tilde{\nu}_I}^2) + |\mu|^2 + \sum_I \epsilon_I^2 + m_{H^2}^2 \\
&= \frac{g^2 + g'^2}{4}\upsilon_2^2 + B\mu\frac{\upsilon_1}{\upsilon_2} - \sum_I B_I\epsilon_I\frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_2}, \\
r_{33} &= \frac{g^2 + g'^2}{8}(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2 + 2\upsilon_{\tilde{\nu}_e}^2) + \epsilon_1^2 + m_{L^1}^2 \\
&= \frac{g^2 + g'^2}{4}\upsilon_{\tilde{\nu}_e}^2 + \mu\epsilon_1\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_e}} - B_1\epsilon_1\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_e}} - \epsilon_1\epsilon_2\frac{\upsilon_{\tilde{\nu}_\mu}}{\upsilon_{\tilde{\nu}_e}} - \epsilon_1\epsilon_3\frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_{\tilde{\nu}_e}}, \\
r_{44} &= \frac{g^2 + g'^2}{8}(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2 + 2\upsilon_{\tilde{\nu}_\mu}^2) + \epsilon_2^2 + m_{L^2}^2 \\
&= \frac{g^2 + g'^2}{4}\upsilon_{\tilde{\nu}_\mu}^2 + \mu\epsilon_2\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_\mu}} - B_2\epsilon_2\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_\mu}} - \epsilon_1\epsilon_2\frac{\upsilon_{\tilde{\nu}_e}}{\upsilon_{\tilde{\nu}_\mu}} - \epsilon_2\epsilon_3\frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_{\tilde{\nu}_\mu}}, \\
r_{55} &= \frac{g^2 + g'^2}{8}(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2 + 2\upsilon_{\tilde{\nu}_\tau}^2) + \epsilon_3^2 + m_{L^3}^2 \\
&= \frac{g^2 + g'^2}{4}\upsilon_{\tilde{\nu}_\tau}^2 + \mu\epsilon_3\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_\tau}} - B_3\epsilon_3\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_\tau}} - \epsilon_1\epsilon_3\frac{\upsilon_{\tilde{\nu}_e}}{\upsilon_{\tilde{\nu}_\tau}} - \epsilon_2\epsilon_3\frac{\upsilon_{\tilde{\nu}_\mu}}{\upsilon_{\tilde{\nu}_\tau}}
\end{aligned} \tag{16}$$

and

$$
\begin{aligned}
e_{12} &= \frac{g^2+g'^2}{4}\upsilon_1\upsilon_2, & e_{13} &= \frac{g^2+g'^2}{4}\upsilon_1\upsilon_{\tilde{\nu}_e}, \\
e_{14} &= \frac{g^2+g'^2}{4}\upsilon_1\upsilon_{\tilde{\nu}_\mu}, & e_{15} &= \frac{g^2+g'^2}{4}\upsilon_1\upsilon_{\tilde{\nu}_\tau}, \\
e_{23} &= \frac{g^2+g'^2}{4}\upsilon_2\upsilon_{\tilde{\nu}_e}, & e_{24} &= \frac{g^2+g'^2}{4}\upsilon_2\upsilon_{\tilde{\nu}_\mu}, \\
e_{25} &= \frac{g^2+g'^2}{4}\upsilon_2\upsilon_{\tilde{\nu}_\tau}, & e_{34} &= \frac{g^2+g'^2}{4}\upsilon_{\tilde{\nu}_e}\upsilon_{\tilde{\nu}_\mu}, \\
e_{35} &= \frac{g^2+g'^2}{4}\upsilon_{\tilde{\nu}_e}\upsilon_{\tilde{\nu}_\tau}, & e_{45} &= \frac{g^2+g'^2}{4}\upsilon_{\tilde{\nu}_\mu}\upsilon_{\tilde{\nu}_\tau}.
\end{aligned}
\tag{17}
$$

In order to obtain the above mass matrix, the Eq. (12) is used. The physical CP-even Higgs can be obtained by

$$
H_i^0 = \sum_{j=1}^{5} Z_{even}^{ij}\chi_j^0 \tag{18}
$$

where $Z_{i,j}^{even}$ $(i,\ j = 1,\ 2,\ 3,\ 4,\ 5)$ are the elements of the matrix that converts the mass matrix Eq. (15) into a diagonal one i.e. translates the current fields into physical fields (corresponding to the eigenstates of the mass matrix).

In the current basis $\Phi_{odd}^T = (\phi_1^0,\ \phi_2^0,\ \phi_{\tilde{\nu}_e}^0,\ \phi_{\tilde{\nu}_\mu}^0,\ \phi_{\tilde{\nu}_\tau}^0)$, the mass matrix for the CP-odd scalar fields can be written as:

$$
\mathcal{M}_{odd}^2 = \begin{pmatrix}
s_{11} & B\mu & -\mu\epsilon_1 & -\mu\epsilon_2 & -\mu\epsilon_3 \\
B\mu & s_{22} & -B_1\epsilon_1 & -B_2\epsilon_2 & -B_3\epsilon_3 \\
-\mu\epsilon_1 & -B_1\epsilon_1 & s_{33} & \epsilon_1\epsilon_2 & \epsilon_1\epsilon_3 \\
-\mu\epsilon_2 & -B_2\epsilon_2 & \epsilon_1\epsilon_2 & s_{44} & \epsilon_2\epsilon_3 \\
-\mu\epsilon_3 & -B_3\epsilon_3 & \epsilon_1\epsilon_3 & \epsilon_2\epsilon_3 & s_{55}
\end{pmatrix} \tag{19}
$$

with

$$
\begin{aligned}
s_{11} &= \frac{g^2+g'^2}{8}(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2) + \mu^2 + m_{H^1}^2 \\
&= \sum_I \mu\epsilon_I \frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_1} + B\mu\frac{\upsilon_2}{\upsilon_1},
\end{aligned}
$$

$$
\begin{aligned}
s_{22} &= -\frac{g^2+g'^2}{8}(\upsilon_1^2-\upsilon_2^2+\upsilon_{\tilde{\nu}_I}^2)+\mu^2+\sum_I \epsilon_I^2+m_{H^2}^2 \\
&= B\mu\frac{\upsilon_1}{\upsilon_2}-\sum_I B_I\epsilon_I\frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_2}, \\
s_{33} &= \frac{g^2+g'^2}{8}(\upsilon_1^2-\upsilon_2^2+\sum_I \upsilon_{\tilde{\nu}_I}^2)+\epsilon_1^2+m_{L^1}^2 \\
&= \mu\epsilon_1\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_e}}-B_1\epsilon_1\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_e}}-\epsilon_1\epsilon_2\frac{\upsilon_{\tilde{\nu}_\mu}}{\upsilon_{\tilde{\nu}_e}}-\epsilon_1\epsilon_3\frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_{\tilde{\nu}_e}}, \\
s_{44} &= \frac{g^2+g'^2}{8}(\upsilon_1^2-\upsilon_2^2+\sum_I \upsilon_{\tilde{\nu}_I}^2)+\epsilon_2^2+m_{L^2}^2 \\
&= \mu\epsilon_2\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_\mu}}-B_2\epsilon_2\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_\mu}}-\epsilon_1\epsilon_2\frac{\upsilon_{\tilde{\nu}_e}}{\upsilon_{\tilde{\nu}_\mu}}-\epsilon_2\epsilon_3\frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_{\tilde{\nu}_\mu}}, \\
s_{55} &= \frac{g^2+g'^2}{8}(\upsilon_1^2-\upsilon_2^2+\sum_I \upsilon_{\tilde{\nu}_I}^2)+\epsilon_3^2+m_{L^3}^2 \\
&= \mu\epsilon_3\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_\tau}}-B_3\epsilon_3\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_\tau}}-\epsilon_1\epsilon_3\frac{\upsilon_{\tilde{\nu}_e}}{\upsilon_{\tilde{\nu}_\tau}}-\epsilon_2\epsilon_3\frac{\upsilon_{\tilde{\nu}_\mu}}{\upsilon_{\tilde{\nu}_\tau}}. \qquad (20)
\end{aligned}
$$

From Eq. (19) and Eq. (20), one can find that the neutral Goldstone boson (with zero-mass) can be given as [11]:

$$
\begin{aligned}
G^0 &\equiv H_6^0 \\
&= \sum_{i=1}^5 Z_{odd}^{1,i}\phi_i^0 \\
&= \frac{1}{\upsilon}(\upsilon_1\phi_1^0-\upsilon_2\phi_2^0+\upsilon_{\tilde{\nu}_e}\phi_{\tilde{\nu}_e}^0+\upsilon_{\tilde{\nu}_\mu}\phi_{\tilde{\nu}_\mu}^0+\upsilon_{\tilde{\nu}_\tau}\phi_{\tilde{\nu}_\tau}^0), \qquad (21)
\end{aligned}
$$

which is indispensable for spontaneous breaking the EW gauge symmetry. Here the $\upsilon = \sqrt{\upsilon_1^2+\upsilon_2^2+\sum\limits_I \upsilon_{\tilde{\nu}_I}^2}$ and the mass of Z-boson $M_Z = \frac{\sqrt{g^2+g'^2}}{2}\upsilon$ is the same as R-parity conserved MSSM. The other four physical neutral bosons can be written as:

$$
H_{5+i}^0(i=2,3,4,5) = \sum_{j=1}^5 Z_{odd}^{i,j}\phi_j^0 \qquad (22)
$$

where $Z_{i,j}^{odd}$ (i, j = 1, 2, 3, 4, 5) is the matrix that converts the current fields into the physical eigenstates.

From the eigenvalue equations, one can find two independent relations:

$$
\sum_{i=1}^5 m_{H_i^0}^2 = \sum_{i=2}^5 m_{H_{5+i}^0}^2 + m_Z^2,
$$

$$\prod_{i=1}^{5} m_{H_i^0}^2 = \left[\frac{\upsilon_1^2 - \upsilon_2^2 + \sum\limits_{I=1}^{3} \upsilon_{\nu_I}^2}{\upsilon^2}\right]^2 m_Z^2 \prod_{i=2}^{5} m_{H_{5+i}^0}^2 . \tag{23}$$

If we introduce the following notations:

$$\begin{aligned}
&\upsilon_1 = \upsilon \cos\beta \cos\theta_\upsilon \\
&\upsilon_2 = \upsilon \sin\beta \\
&\sqrt{\sum_{I=1}^{3} \upsilon_{\nu_I}^2} = \upsilon \cos\beta \sin\theta_\upsilon
\end{aligned} \tag{24}$$

the second relation of Eq. (23) can be written as:

$$\prod_{i=1}^{5} m_{H_i^0}^2 = \cos^2 2\beta m_Z^2 \prod_{i=2}^{5} m_{H_{5+i}^0}^2 \tag{25}$$

The first relation of Eq. (23) is also obtained in Ref [12], whereas we consider the second relation of Eq. (23) is also important, the two equations are independent restrictions on the masses of neutral Higgs bosons. For instance, from Eq. (23) and Eq. (25), we have a upper limit on the mass of the lightest Higgs at tree level in the model:

$$m_{H_1^0}^2 \leq m_{H_n^0}^2 \left(\frac{m_Z^2 \cos^2 2\beta}{m_{H_n^0}^2}\right)^{\frac{1}{n-1}} \frac{1 - \frac{1}{n-1}\frac{m_Z^2}{m_{H_n^0}^2}}{1 - \frac{1}{n-1}\left(\frac{m_Z^2 \cos^2 2\beta}{m_{H_n^0}^2}\right)^{\frac{1}{n-1}}} \tag{26}$$

where $n \geq 2$ is the number of the CP-even Higgs, $m_{H_1^0}$ is the mass of the lightest one among them and $m_{H_n^0}$ is the heaviest one. Some points should be noted about Eq. (26):

- From Eq. (23) and Eq. (25), we can find $m_{H_n^0}^2 \geq m_z^2$.
- When $n = 2$ or $m_{H_1^0}^2 = \cdots = m_{H_n^0}^2 = m_{H_{n+2}^0}^2 = \cdots = m_{H_{n+n}^0}^2 = m_Z^2$, $\cos^2 2\beta = 1$, "=" is established.
- In the case of MSSM with R-parity (n=2), $m_{H_1^0} = m_Z^2 \cos^2 2\beta \frac{1-\frac{m_Z^2}{m_{H_2^0}^2}}{1-\frac{m_Z^2}{m_{H_2^0}^2}\cos^2 2\beta} \leq m_Z^2 \cos^2 2\beta$ is recovered.

So when $n > 2$, we cannot imposed a upper limit on the $m_{H_1^0}$ as that for the R-parity conserved MSSM at the tree level, namely, for the later it is just the case n=2 [19]:

$$m_{H_1^0}^2 \leq m_Z^2 \cos^2 2\beta \leq m_Z^2. \tag{27}$$

Considering experimental data, one cannot rule out large ϵ_I (I=1, 2, 3) [21,29]. Further more, even if $\epsilon_I \ll \mu$, we still have no reason to assume $B_I\epsilon_I \ll B\mu$ in general case. In the MSSM with R-parity, the radiative corrections to mass of the lightest Higgs are large [26], when complete one-loop corrections and leading two-loop corrections of $\mathcal{O}(\alpha\alpha_s)$ are included, the Ref [27] gives the limit on the lightest Higgs mass: $m_{H_1^0} \leq 132$GeV. In the MSSM without R-parity, there is not so stringent restriction on the lightest Higgs even at the tree level.

With the "current" basis $\Phi_c = (H_2^{1*}, H_1^2, \tilde{L}_2^{1*}, \tilde{L}_2^{2*}, \tilde{L}_2^{3*}, \tilde{R}^1, \tilde{R}^2, \tilde{R}^3)$ and Eq. (6), we can find the following mass terms in Lagrangian:

$$\mathcal{L}_m^C = -\Phi_c^\dagger \mathcal{M}_c^2 \Phi_c \tag{28}$$

and $\mathcal{M}_c^2$ is given in Appendix.A. Diagonalizing the mass matrix for the charged Higgs bosons, we obtain the zero mass Goldstone boson state:

$$\begin{aligned} H_1^+ &= \sum_{i=1}^{8} Z_c^{1,i} \Phi_c^i \\ &= \frac{1}{\upsilon}(\upsilon_1 H_2^{1*} - \upsilon_2 H_1^2 + \upsilon_{\tilde{\nu}_e} \tilde{L}_2^{1*} + \upsilon_{\tilde{\nu}_\mu} \tilde{L}_2^{2*} + \upsilon_{\tilde{\nu}_\tau} \tilde{L}_2^{3*}), \end{aligned} \tag{29}$$

together with the charge conjugate state H_1^-, which are indispensable to break electroweak symmetry and give $W^\pm$ bosons masses. With the transformation matrix Z_c^{ij} (converts from the current fields into the physical eigenstates basis), the other seven physical eigenstates H_i^+ ($i = 2, 3, 4, 5, 6, 7, 8$) can be expressed as:

$$H_i^+ = \sum_{j=1}^{8} Z_c^{i,j} \Phi_j^c \quad (i, j = 1, \cdots, 8). \tag{30}$$

B. The mixing of neutralinos with neutrino:

Due to the lepton number violation in the MSSM without R-parity, fresh and interesting mixing of neutralinos with neutrinos and charginos with charged leptons may happen. The piece of Lagrangian responsible for the mixing of neutralinos with neutrinos is:

$$\mathcal{L}^{mass}_{\kappa_i^0} = \{ig\frac{1}{\sqrt{2}}\tau^i_{ij}\lambda^i_A\psi_j A^*_i + ig'\sqrt{2}Y_i\lambda_B\psi_i A^*_i - \frac{1}{2}\frac{\partial^2\mathcal{W}}{\partial A_i\partial A_j}\psi_i\psi_j + h.c.\}$$
$$+m_1(\lambda_B\lambda_B + h.c.) + m_2(\lambda^i_A\lambda^i_A + h.c.) \tag{31}$$

where $\mathcal{W}$ is given by Eq. (4). $\tau^i/2$ and Y_i are the generators of the SU(2)×U(1) gauge group and ψ_i, A_i stand for generic two-component fermions and scalar fields. Writing down the Eq. (31) explicitly, we obtain:

$$\mathcal{L}^{mass}_{\chi_i^0} = -\frac{1}{2}(\Phi^0)^T\mathcal{M}_N\Phi^0 + h.c. \tag{32}$$

with the current basis $(\Phi^0)^T = (-i\lambda_B,\ -i\lambda^3_A,\ \psi^1_{H^1},\ \psi^2_{H^2},\ \nu_{e_L},\ \nu_{\mu_L},\ \nu_{\tau_L})$ and

$$\mathcal{M}_N = \begin{pmatrix} 2m_1 & 0 & -\frac{1}{2}g'\upsilon_1 & \frac{1}{2}g'\upsilon_2 & -\frac{1}{2}g'\upsilon_{\tilde{\nu}_e} & -\frac{1}{2}g'\upsilon_{\tilde{\nu}_\mu} & -\frac{1}{2}g'\upsilon_{\tilde{\nu}_\tau} \\ 0 & 2m_2 & \frac{1}{2}g\upsilon_1 & -\frac{1}{2}g\upsilon_2 & \frac{1}{2}g\upsilon_{\tilde{\nu}_e} & \frac{1}{2}g\upsilon_{\tilde{\nu}_\mu} & \frac{1}{2}g\upsilon_{\tilde{\nu}_\tau} \\ -\frac{1}{2}g'\upsilon_1 & \frac{1}{2}g\upsilon_1 & 0 & -\frac{1}{2}\mu & 0 & 0 & 0 \\ \frac{1}{2}g'\upsilon_2 & -\frac{1}{2}g\upsilon_2 & -\frac{1}{2}\mu & 0 & \frac{1}{2}\epsilon_1 & \frac{1}{2}\epsilon_2 & \frac{1}{2}\epsilon_3 \\ -\frac{1}{2}g'\upsilon_{\tilde{\nu}_e} & \frac{1}{2}g\upsilon_{\tilde{\nu}_e} & 0 & \frac{1}{2}\epsilon_1 & 0 & 0 & 0 \\ -\frac{1}{2}g'\upsilon_{\tilde{\nu}_\mu} & \frac{1}{2}g\upsilon_{\tilde{\nu}_\mu} & 0 & \frac{1}{2}\epsilon_2 & 0 & 0 & 0 \\ -\frac{1}{2}g'\upsilon_{\tilde{\nu}_\tau} & \frac{1}{2}g\upsilon_{\tilde{\nu}_\tau} & 0 & \frac{1}{2}\epsilon_3 & 0 & 0 & 0 \end{pmatrix} \tag{33}$$

The mixing has the formulation:

$$\begin{aligned} -i\lambda_B &= Z_N^{1i}\tilde{\chi}^0_i, & -i\lambda^3_A &= Z_N^{2i}\tilde{\chi}^0_i, \\ \psi_{H^1_1} &= Z_N^{3i}\tilde{\chi}^0_i, & \psi_{H^2_2} &= Z_N^{4i}\tilde{\chi}^0_i, \\ \nu_{e_L} &= Z_N^{5i}\tilde{\chi}^0_i, & \nu_{\mu_L} &= Z_N^{6i}\tilde{\chi}^0_i, \\ \nu_{\tau_L} &= Z_N^{7i}\tilde{\chi}^0_i \end{aligned} \tag{34}$$

and transformation matrix Z_N has the property

$$Z_N^T\mathcal{M}_N Z_N = diag(m_{\tilde{\kappa}^0_1}, m_{\tilde{\kappa}^0_2}, m_{\tilde{\kappa}^0_3}, m_{\tilde{\kappa}^0_4}, m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}). \tag{35}$$

For convenience, we formulate all the neutral fermions into four component spinors as the follows:

$$\nu_e = \begin{pmatrix} \tilde{\chi}_5^0 \\ \bar{\tilde{\chi}}_5^0 \end{pmatrix} \tag{36}$$

$$\nu_\mu = \begin{pmatrix} \tilde{\chi}_6^0 \\ \bar{\tilde{\chi}}_6^0 \end{pmatrix} \tag{37}$$

$$\nu_\tau = \begin{pmatrix} \tilde{\chi}_7^0 \\ \bar{\tilde{\chi}}_7^0 \end{pmatrix} \tag{38}$$

$$\kappa_i^0(i=1,2,3,4) = \begin{pmatrix} \tilde{\chi}_i^0 \\ \bar{\tilde{\chi}}_i^0 \end{pmatrix} \tag{39}$$

From Eq. (33), we find that only one type neutrinos obtains mass from the mixing [28], we can assume it is the τ-neutrino. One of the stringent restrictions comes from the bound that the mass of τ-neutrino should be less than 20 MeV [14]. For convenience, sometimes we will call the mixing of neutralinos and neutrinos as neutralinos shortly late on.

C. The mixing of charginos with charged leptons

Similar to the mixing of neutralinos and neutrinos, charginos mix with the charged leptons and form a set of charged fermions: e^-, μ^-, τ^-, $\kappa_1^\pm$, $\kappa_2^\pm$. In current basis, $\Psi^{+T} = (-i\lambda^+, \tilde{H}_2^1, e_R^+, \mu_R^+, \tau_R^+)$ and $\Psi^{-T} = (-i\lambda^-, \tilde{H}_1^2, e_L^-, \mu_L^-, \tau_L^-)$, the charged fermion mass terms in the Lagrangian can be written as [15]

$$\mathcal{L}_{\chi_i^\pm}^{mass} = -\Psi^{-T}\mathcal{M}_C\Psi^+ + h.c. \tag{40}$$

and the mass matrix:

$$\mathcal{M}_C = \begin{pmatrix} 2m_2 & \frac{ev_2}{\sqrt{2}S_W} & 0 & 0 & 0 \\ \frac{ev_1}{\sqrt{2}S_W} & \mu & \frac{l_1 v_{\tilde{\nu}_e}}{\sqrt{2}} & \frac{l_2 v_{\tilde{\nu}_\mu}}{\sqrt{2}} & \frac{l_3 v_{\tilde{\nu}_\tau}}{\sqrt{2}} \\ \frac{ev_{\tilde{\nu}_e}}{\sqrt{2}S_W} & \epsilon_1 & \frac{l_1 v_1}{\sqrt{2}} & 0 & 0 \\ \frac{ev_{\tilde{\nu}_\mu}}{\sqrt{2}S_W} & \epsilon_2 & 0 & \frac{l_2 v_1}{\sqrt{2}} & 0 \\ \frac{ev_{\tilde{\nu}_\tau}}{\sqrt{2}S_W} & \epsilon_3 & 0 & 0 & \frac{l_3 v_1}{\sqrt{2}} \end{pmatrix}. \tag{41}$$

Here $S_W = \sin\theta_W$ and $\lambda^{\pm} = \frac{\lambda_A^1 \mp i\lambda_A^2}{\sqrt{2}}$. Two mixing matrices Z_+, Z_- can be obtained by diagonalizing the mass matrix $\mathcal{M}_c$ i.e. the product $(Z_+)^T\mathcal{M}_C Z_-$ is diagonal matrix:

$$(Z_+)^T\mathcal{M}_C Z_- = \begin{pmatrix} m_{\kappa_1^-} & 0 & 0 & 0 & 0 \\ 0 & m_{\kappa_2^-} & 0 & 0 & 0 \\ 0 & 0 & m_e & 0 & 0 \\ 0 & 0 & 0 & m_\mu & 0 \\ 0 & 0 & 0 & 0 & m_\tau \end{pmatrix} \tag{42}$$

If we denote the mass eigenstates with $\tilde{\chi}$:

$$\begin{aligned} &-i\lambda_A^{\pm} = Z_{\pm}^{1i}\tilde{\chi}_i^{\pm}, && \psi_{H^2}^1 = Z_+^{2i}\tilde{\chi}_i^+, \\ &\psi_{H^1}^2 = Z_-^{2i}\tilde{\chi}_i^-, && e_L = Z_-^{3i}\tilde{\chi}_i^-, \\ &e_R = Z_+^{3i}\tilde{\chi}_i^+, && \mu_L = Z_-^{4i}\tilde{\chi}_i^-, \\ &\mu_R = Z_+^{4i}\tilde{\chi}_i^+, && \tau_L = Z_-^{5i}\tilde{\chi}_i^-, \\ &\tau_R = Z_+^{5i}\tilde{\chi}_i^+. \end{aligned} \tag{43}$$

The four-component fermions are defined as:

$$\kappa_i^+(i = 1, 2, 3, 4, 5) = \begin{pmatrix} \tilde{\chi}_i^+ \\ \bar{\tilde{\chi}}_i^- \end{pmatrix} \tag{44}$$

where $\kappa_1^{\pm}$, $\kappa_2^{\pm}$ are the usual charginos and $\kappa_i^{\pm}$ $(i = 3,\, 4,\, 5)$ correspond to e, μ and τ leptons respectively. For convenience, sometimes we will call the mixing of charginos with charged leptons as charginos shortly late on.

From the above analyses, we have achieved the mass spectrum of the neutralino - neutrinos, chargino - charged leptons, neutral Higgs - sneutrinos and charged Higgs - charged sleptons. For the interaction vertices are also important, thus we will give the Feynman rules which are different from those of the MSSM with R-parity in next section.

III. FEYNMAN RULES FOR THE R-PARITY VIOLATING INTERACTION

We have discussed the mass spectrum of the MSSM with bilinear R-parity violation. Now, we are discussing the Feynman rules for the model that are different from those in MSSM with R-parity. We are working in the t′Hooft- Feynman gauge [16] which has the gauge fixed terms as:

$$\begin{aligned}
\mathcal{L}_{GF} &= -\frac{1}{2\xi}\Big(\partial^\mu A^3_\mu + \xi M_Z C_W H^0_6\Big)^2 - \frac{1}{2\xi}\Big(\partial^\mu B_\mu - \xi M_Z S_W H^0_6\Big)^2 - \frac{1}{2\xi}\Big(\partial^\mu A^1_\mu \\
&\quad + \frac{i}{\sqrt{2}}\xi M_W\Big(H^+_1 - H^-_1\Big)\Big)^2 - \frac{1}{2\xi}\Big(\partial^\mu A^2_\mu + \frac{1}{\sqrt{2}}\xi M_W\Big(H^+_1 + H^-_1\Big)\Big)^2 \\
&= \Big\{ - \frac{1}{2\xi}(\partial^\mu Z_\mu)^2 - \frac{1}{2\xi}(\partial^\mu F_\mu)^2 - \frac{1}{\xi}(\partial^\mu W^+_\mu)(\partial^\mu W^-_\mu)\Big\} - \Big\{M_Z H^0_6 \partial^\mu Z_\mu \\
&\quad + iM_W\Big(H^+_1\partial^\mu W^-_\mu - H^-_1\partial^\mu W^+_\mu\Big)\Big\} - \Big\{\frac{1}{2}\xi M^2_Z H^{0^2}_6 - \xi M^2_W H^+_1 H^-_1\Big\},
\end{aligned} \tag{45}$$

where $C_W = \cos\theta_W$ and H^0_6, $H^\pm_1$ were given as Eq. (21) and Eq. (29). By inserting Eq. (45) into interaction Lagrangian, one obtains the desired vertices for the Higgs bosons. If CP is conserved i.e. we assume the relevant parameters are real, one finds (by analyzing the $H^0_i f\bar{f}$ couplings) that H^0_1, H^0_2, H^0_3 H^0_4, H^0_5 are scalars and H^0_6, H^0_7, H^0_8, H^0_9, H^0_{10} are pseudoscalar.

A. Feynman rules for Higgs (slepton)- gauge boson interactions

Let us compute the vertices of Higgs (slepton)- gauge bosons in the model. The original interaction terms of Higgs bosons and gauge bosons are given as

$$\begin{aligned}
\mathcal{L}^1_{int} &= -\sum_I(\mathcal{D}_\mu\tilde{L}^{I\dagger}\mathcal{D}^\mu\tilde{L}^I - \mathcal{D}_\mu\tilde{R}^{I*}\mathcal{D}^\mu\tilde{R}^I) - \mathcal{D}_\mu H^{1\dagger}\mathcal{D}^\mu H^1 - \mathcal{D}_\mu H^{2\dagger}\mathcal{D}^\mu H^2 \\
&= \sum_I\Big\{\Big[i\tilde{L}^{I\dagger}\Big(g\frac{\tau^i}{2}A^i_\mu - \frac{1}{2}g'B_\mu\Big)\partial^\mu\tilde{L}^I + h.c.\Big] - \tilde{L}^{I\dagger}\Big(g\frac{\tau^i}{2}A^i_\mu \\
&\quad -\frac{1}{2}g'B_\mu\Big)\Big(g\frac{\tau^j}{2}A^{j\mu} - \frac{1}{2}g'B^\mu\Big)\tilde{L}^I + \Big(ig'B_\mu\tilde{R}^{I*}\partial^\mu\tilde{R}^I \\
&\quad +h.c.\Big) - g'^2\tilde{R}^{I*}\tilde{R}^I B_\mu B^\mu\Big\} + \Big\{H^{1\dagger}\Big(g\frac{\tau^i}{2}A^i_\mu - \frac{1}{2}g'B_\mu\Big)\partial^\mu H^1 \\
&\quad +h.c.\Big\} - H^{1\dagger}\Big(g\frac{\tau^i}{2}A^i_\mu - \frac{1}{2}g'B_\mu\Big)\Big(g\frac{\tau^j}{2}A^{j\mu} - \frac{1}{2}g'B^\mu\Big)H^1
\end{aligned}$$

$$
\begin{aligned}
&+\Big\{H^{2\dagger}\Big(g\frac{\tau^i}{2}A^i_\mu-\frac{1}{2}g'B_\mu\Big)\partial^\mu H^2+h.c.\Big\}-H^{2\dagger}\Big(g\frac{\tau^i}{2}A^i_\mu\\
&-\frac{1}{2}g'B_\mu\Big)\Big(g\frac{\tau^j}{2}A^{j\mu}-\frac{1}{2}g'B^\mu\Big)H^2\\
=\;&\mathcal{L}_{SSV}+\mathcal{L}_{SVV}+\mathcal{L}_{SSVV}. \qquad (46)
\end{aligned}
$$

Here $\mathcal{L}_{SSV}$, $\mathcal{L}_{SVV}$ and $\mathcal{L}_{SSVV}$ represent the interactions in the physical basis, thus we have

$$
\begin{aligned}
\mathcal{L}_{SSV}=&\frac{i}{2}\sqrt{g^2+g'^2}Z_\mu\Big\{\partial^\mu\phi^0_1\chi^0_1-\phi^0_1\partial^\mu\chi^0_1-\partial^\mu\phi^0_2\chi^0_2+\phi^0_2\partial^\mu\chi^0_2\\
&+\sum_I\Big(\partial^\mu\phi^0_{\tilde{\nu}_I}\chi^0_{\tilde{\nu}_I}-\phi^0_{\tilde{\nu}_I}\partial^\mu\chi^0_{\tilde{\nu}_I}\Big)\Big\}+\frac{1}{2}g\Big\{W^+_\mu\Big[\chi^0_1\partial^\mu H^1_2-\partial^\mu\chi^0_1H^1_2\\
&-\chi^0_2\partial^\mu H^{2*}_1+\partial^\mu\chi^0_2H^{2*}_1+\sum_I\chi^0_{\tilde{\nu}_I}\partial^\mu\tilde{L}^I_2-\partial^\mu\chi^0_{\tilde{\nu}_I}\tilde{L}^I_2\Big]+h.c.\Big\}\\
&+\frac{i}{2}g\Big\{W^+_\mu\Big[\phi^0_1\partial^\mu H^{1*}_2-\partial^\mu\phi^0_1H^1_2+\phi^0_2\partial^\mu H^{2*}_1-\partial^\mu\phi^0_2H^{2*}_1\\
&+\sum_I(\phi^0_{\tilde{\nu}_I}\partial^\mu\tilde{L}^I_2-\partial^\mu\phi^0_{\tilde{\nu}_I}\tilde{L}^I_2)\Big]-h.c.\Big\}+\Big\{\frac{1}{2}\sqrt{g^2+g'^2}\Big(\cos2\theta_WZ_\mu\\
&-\sin2\theta_WA_\mu\Big)\Big[\sum_I\Big(\tilde{L}^{I*}_2\partial^\mu\tilde{L}^I_2-\partial^\mu\tilde{L}^{I*}_2\tilde{L}^I_2\Big)-H^{2*}_1\partial^\mu H^2_1\\
&+\partial^\mu H^{2*}_1H^2_1+H^{1*}_2\partial^\mu H^1_2-\partial^\mu H^{1*}_2H^1_2\Big]+\Big(2\sin^2\theta_WZ_\mu\\
&+2\sin\theta_W\cos\theta_WA_\mu\Big)\Big[\sum_I\Big(\tilde{R}^{I*}\partial^\mu\tilde{R}^I-\partial^\mu\tilde{R}^{I*}\tilde{R}^I\Big)\Big]\Big\}\\
=&\frac{i}{2}\sqrt{g^2+g'^2}C^{ij}_{eo}\Big(\partial^\mu H^0_{5+i}H^0_j-H^0_{5+i}\partial^\mu H^0_j\Big)Z_\mu\\
&+\Big\{\frac{1}{2}gC^{ij}_{ec}\Big(H^0_i\partial^\mu H^-_j-\partial^\mu H^0_iH^-_j\Big)W^+_\mu+h.c.\Big\}\\
&+\Big\{\frac{i}{2}gC^{ij}_{co}\Big(H^0_{5+i}\partial^\mu H^-_j-\partial^\mu H^0_{5+i}H^-_j\Big)W^+_\mu\\
&+h.c.\Big\}+\Big\{\frac{1}{2}\sqrt{g^2+g'^2}\Big[\Big(\cos2\theta_W\delta^{ij}\\
&-C^{ij}_c\Big)Z_\mu\Big(H^-_i\partial^\mu H^+_j-\partial^\mu H^-_iH^+_j\Big)\\
&-\sin2\theta_WA_\mu\Big(H^-_i\partial^\mu H^+_i-\partial^\mu H^-_iH^+_i\Big)\Big]\Big\}, \qquad (47)
\end{aligned}
$$

with

$$
C^{ij}_{eo}=\sum_{\alpha=1}^{5}Z^{i,\alpha}_{odd}Z^{j,\alpha}_{even}-2Z^{i,2}_{odd}Z^{j,2}_{even},
$$

$$
\begin{aligned}
C_{ec}^{ij} &= \sum_{\alpha=1}^{5} Z_{even}^{i,\alpha} Z_c^{j,\alpha} - 2 Z_{even}^{i,2} Z_c^{j,2}, \\
C_{co}^{ij} &= \sum_{\alpha=1}^{5} Z_{odd}^{i,\alpha} Z_c^{j,\alpha} - 2 Z_{odd}^{i,2} Z_c^{j,2}, \\
C_{c}^{ij} &= \sum_{\alpha=6}^{8} Z_{c}^{i,\alpha} Z_c^{j,\alpha}.
\end{aligned} \tag{48}
$$

Where the transformation matrices Z_{even}, Z_{odd} and Z_c are defined in section II.

$$
\begin{aligned}
\mathcal{L}_{SVV} =& \frac{g^2+g'^2}{4}\Big(\upsilon_1\chi_1^0+\upsilon_2\chi_2^0+\sum_I \upsilon_{\tilde{\nu}_I}\chi_{\tilde{\nu}_I}^0\Big)\Big(Z_\mu Z^\mu + 2\cos^2\theta_W W_\mu^- W_+^\mu\Big) \\
&+\bigg\{\frac{g^2+g'^2}{4}\Big[\cos\theta_W\Big(-1+\cos 2\theta_W\Big)Z_\mu W_+^\mu\Big(\upsilon_1 H_2^1-\upsilon_2 H_1^{2*} \\
&+\sum_I \upsilon_{\tilde{\nu}_I}\tilde{L}_2^I\Big) - \cos\theta_W \sin 2\theta_W A_\mu W^{+\mu}\Big(\upsilon_1 H_2^1-\upsilon_2 H_1^{2*} \\
&+\sum_I \upsilon_{\tilde{\nu}_I}\tilde{L}_2^I\Big)\Big] + h.c.\bigg\} \\
=& \frac{g^2+g'^2}{4} C_{even}^i\Big(H_i^0 Z_\mu Z^\mu + 2\cos^2\theta_W H_i^0 W_\mu^- W^{+\mu}\Big) \\
&-\frac{g^2+g'^2}{2} S_W C_W \upsilon\Big[S_W Z_\mu W^{+\mu} H_1^- + C_W A_\mu W^{+\mu} H_1^- + h.c.\Big],
\end{aligned} \tag{49}
$$

with

$$
C_{even}^i = Z_{even}^{i,1}\upsilon_1 + Z_{even}^{i,2}\upsilon_2 + \sum_I Z_{even}^{i,I+2}\upsilon_{\tilde{\nu}_I} \tag{50}
$$

The piece of $\mathcal{L}_{SSVV}$ is given as

$$
\begin{aligned}
\mathcal{L}_{SSVV} =& -\frac{g^2+g'^2}{4}\Big[\frac{1}{2}\Big(\chi_1^0\chi_1^0+\chi_2^0\chi_2^0+\sum_I \chi_{\tilde{\nu}_I}^0\chi_{\tilde{\nu}_I}^0\Big)Z_\mu Z^\mu \\
&+\cos^2\theta_W\Big(\chi_1^0\chi_1^0+\chi_2^0\chi_2^0+\sum_I \chi_{\tilde{\nu}_I}^0\chi_{\tilde{\nu}_I}^0\Big)W_\mu^- W^{+\mu}\Big] - \frac{g^2+g'^2}{4}\Big[\frac{1}{2}\Big(\phi_1^0\phi_1^0 \\
&+\phi_2^0\phi_2^0+\sum_I \phi_{\tilde{\nu}_I}^0\phi_{\tilde{\nu}_I}^0\Big)Z_\mu Z^\mu + \cos^2\theta_W\Big(\phi_1^0\phi_1^0+\phi_2^0\phi_2^0+\phi_{\tilde{\nu}_I}^0\phi_{\tilde{\nu}_I}^0\Big)W_\mu^- W^{+\mu}\Big] \\
&-\frac{g^2+g'^2}{4}\cos\theta_W\Big[\Big(-1+\cos 2\theta_W\Big)Z_\mu W^{+\mu}\Big(\chi_1^0 H_2^1-\chi_2^0 H_1^{2*} \\
&+\sum_I \chi_{\tilde{\nu}_I}^0\tilde{L}_2^I\Big) - \cos\theta_W\sin 2\theta_W A_\mu W^{+\mu}\Big(\chi_1^0 H_2^1 - \chi_2^0 H_1^{2*} \\
&+\sum_I \chi_{\tilde{\nu}_I}^0\tilde{L}_2^I\Big) + h.c.\Big] + \frac{i(g^2+g'^2)}{4}\cos\theta_W\Big[\Big(-1+\cos 2\theta_W\Big)Z_\mu W^{+\mu}\Big(\phi_1^0 H_2^1
\end{aligned}
$$

$$
\begin{aligned}
&-\phi_2^0 H_1^{2*} + \sum_I \phi^0_{\tilde{\nu}_I} \tilde{L}_2^I\Big) - \cos\theta_W \sin 2\theta_W A_\mu W^{+\mu}\Big(\phi_1^0 H_2^1 \\
&-\phi_2^0 H_1^{2*} + \sum_I \phi^0_{\tilde{\nu}_I} \tilde{L}_2^I\Big) + h.c.\Big] - \frac{1}{4}(g^2 + g'^2)\Big[\sin^2 2\theta_W A_\mu A^\mu \\
&\Big(H_2^{1*} H_2^1 + H_1^{2*} H_1^2 + \sum_I \tilde{L}_2^{I*} \tilde{L}_2^I\Big) \\
&+ \cos^2 2\theta_W Z_\mu Z^\mu \Big(H_2^{1*} H_2^1 + H_1^{2*} H_1^2 + \sum_I \tilde{L}_2^{I*} \tilde{L}_2^I\Big) \\
&- \sin 4\theta_W Z_\mu A^\mu \Big(H_2^{1*} H_2^1 + H_1^{2*} H_1^2 + \sum_I \tilde{L}_2^{I*} \tilde{L}_2^I\Big) \\
&+2\cos^2\theta_W \Big(H_2^{1*} H_2^1 + H_1^{2*} H_1^2 + \sum_I \tilde{L}_2^{I*} \tilde{L}_2^I\Big)\Big] \\
&- \sum_I g'^2 \tilde{R}^{I*} \tilde{R}^I B_\mu B^\mu \\
= & -\frac{1}{4}(g^2 + g'^2)\Big(\frac{1}{2} H_i^0 H_i^0 Z_\mu Z^\mu + \cos^2\theta_W H_i^0 H_i^0 W_\mu^- W^{+\mu}\Big) \\
&-\frac{1}{4}(g^2 + g'^2)\Big(\frac{1}{2} H_{5+i}^0 H_{5+i}^0 Z_\mu Z^\mu + \cos^2\theta_W H_{5+i}^0 H_{5+i}^0 W_\mu^- W^{+\mu}\Big) \\
&+\frac{1}{4}(g^2 + g'^2)\sin 2\theta_W \Big\{ C_{ec}^{ij}\Big[\sin\theta_W H_i^0 Z_\mu W^{+\mu} H_j^- \\
&+ \cos\theta_W H_i^0 A_\mu W^{+\mu} H_j^-\Big] + h.c.\Big\} \\
&-\frac{i}{4}(g^2 + g'^2)\sin 2\theta_W \Big\{ C_{co}^{ij}\Big[\sin\theta_W H_{5+i}^0 Z_\mu W^{+\mu} H_j^- \\
&+ \cos\theta_W H_{5+i}^0 A_\mu W^{+\mu} H_i^-\Big] - h.c.\Big\} \\
&-\frac{1}{4}(g^2 + g'^2)\Big\{ 2\cos^2\theta_W \Big(\delta_{ij} - C_c^{ij}\Big) H_i^- H_j^+ W_\mu^- W^{+\mu} \\
&+\Big[\cos^2 2\theta_W \delta_{ij} - C_c^{ij}\Big(4\sin^3\theta_W - \cos^2 2\theta_W\Big)\Big] H_i^- H_j^+ Z_\mu Z^\mu \\
&+ \sin^2 2\theta_W \delta_{ij} H_i^- H_j^+ A_\mu A^\mu + \Big[\sin 4\theta_W \delta_{ij} \\
&- C_c^{ij}\Big(\sin 4\theta_W + 8\sin^2\theta_W \cos\theta_W\Big)\Big] Z_\mu A^\mu H_i^- H_j^+ \Big\}\ ,
\end{aligned}
\tag{51}
$$

where the C_{eo}^{ij}, C_{co}^{ij} and C_c^{ij} are defined in Eq. (48). The relevant Feynman rules may be summarized in Fig. 1, Fig. 2, Fig. 3 and Fig. 4. We would emphasize some features about them. First, the presence of the vertices $Z_\mu H_i^0 H_{5+j}^0$ ($i,\ j = 1,\ 2,\ 3,\ 4,\ 5$) and the forbiddance of the vertices $Z_\mu H_i^0 H_j^0$ and $Z_\mu H_{5+i}^0 H_{5+j}^0$ ($i,\ j = 1,\ 2,\ 3,\ 4,\ 5$) couplings are determined by CP nature. Second, besides the $W_\mu^+ Z^\mu H_1^-$ (H_1^- is just the charged Goldstone boson)

interaction, there are not vertices $W_\mu^+ Z^\mu H_i^-$ ($i = 2, 3, 4, 5, 6, 7, 8$) at the tree level, that is the same as the MSSM with R-parity and general two-Higgs doublet models [17].

B. Self-couplings of the Higgs bosons (sleptons)

It is straightforward to insert Eqs. (18, 21, 22, 29, 30) into Eqs. (6) to obtain the desired interaction terms. Similar to the interaction of gauge-Higgs (slepton) bosons, we split the Lagrangian into pieces:

$$\mathcal{L}_{int}^S = \mathcal{L}_{SSS} + \mathcal{L}_{SSSS} \tag{52}$$

where $\mathcal{L}_{SSS}$ represents trilinear coupling terms, and $\mathcal{L}_{SSSS}$ represents four scalar boson coupling terms. The trilinear piece is most interesting. If the masses of the scalars are appropriate, the decays of one Higgs boson into two other Higgs bosons may be opened. After tedious computation, we have:

$$\begin{aligned}\mathcal{L}_{SSS} = & -\frac{g^2+g'^2}{8} A_{even}^{ij} B_{even}^k H_i^0 H_j^0 H_k^0 - \frac{g^2+g'^2}{8} A_{odd}^{ij} B_{even}^k H_{5+i}^0 H_{5+j}^0 H_k^0 \\ & - A_{ec}^{kij} H_k^0 H_i^- H_j^+ + i A_{oc}^{kij} H_{5+k}^0 H_i^- H_j^+ \end{aligned} \tag{53}$$

and

$$\begin{aligned}\mathcal{L}_{SSSS} = & -\frac{g^2+g'^2}{32} A_{even}^{ij} A_{even}^{kl} H_i^0 H_j^0 H_k^0 H_l^0 - \frac{g^2+g'^2}{32} A_{odd}^{ij} A_{odd}^{kl} H_{5+i}^0 H_{5+j}^0 H_{5+k}^0 H_{5+l}^0 \\ & -\frac{g^2+g'^2}{16} A_{even}^{ij} A_{odd}^{kl} H_i^0 H_j^0 H_{5+k}^0 H_{5+l}^0 - \mathcal{A}_{ec}^{klij} H_k^0 H_l^0 H_i^- H_j^+ \\ & - \mathcal{A}_{oc}^{klij} H_{5+k}^0 H_{5+l}^0 H_i^- H_j^+ - i \mathcal{A}_{eoc}^{klij} H_k^0 H_{5+l}^0 H_i^- H_j^+ - \mathcal{A}_{cc}^{klij} H_k^- H_l^+ H_i^- H_j^+ \end{aligned} \tag{54}$$

with

$$\begin{aligned} A_{even}^{ij} &= \sum_{\alpha=1}^{5} Z_{even}^{i,\alpha} Z_{even}^{j,\alpha} - 2 Z_{even}^{i,2} Z_{even}^{j,2} , \\ A_{odd}^{ij} &= \sum_{\alpha=1}^{5} Z_{odd}^{i,\alpha} Z_{odd}^{j,\alpha} - 2 Z_{odd}^{i,2} Z_{odd}^{j,2} , \\ B_{even}^{i} &= \upsilon_1 Z_{even}^{i,1} - \upsilon_2 Z_{even}^{i,2} + \sum_I \upsilon_{\tilde{\nu}_I} Z_{even}^{i,I+2} . \end{aligned} \tag{55}$$

The definitions of A^{kij}_{ec}, A^{kij}_{oc}, $\mathcal{A}^{klij}_{ec}$, $\mathcal{A}^{klij}_{oc}$, $\mathcal{A}^{klij}_{eoc}$ and $\mathcal{A}^{ijkl}_{cc}$ can be found in Appendix.C. The Feynman rules are summarized in Fig. 5 and Fig. 6. Note that the lepton number violation has led to very complicated form for the $\mathcal{L}_{SSS}$ and $\mathcal{L}_{SSSS}$.

C. The couplings of Higgs to charginos (charged leptons) and neutralinos (neutrinos)

In this subsection, we compute the interactions of the Higgs bosons with the supersymmetric partners of the gauge and Higgs bosons (the gauginos and higgsinos). After spontaneous breaking of the gauge symmetry SU(2)×U(1), the gauginos, higgsinos and leptons with the same electric charge will mix as we have described in Section II. Let us proceed now to compute interesting interactions $S\tilde{\kappa}^0_i\tilde{\kappa}^0_j$ (Higgs-neutralinos-neutralinos interactions) etc.

The original interactions (in two-component notations) are [18]:

$$\begin{aligned}
\mathcal{L}_{S\kappa\kappa} = & i\sqrt{2}g\Big(H^{1\dagger}\frac{\tau^i}{2}\lambda^i_A\psi_{H^1} - \bar{\psi}_{H^1}\frac{\tau^i}{2}\bar{\lambda}^i_A H^1\Big) - i\sqrt{2}g'\Big(\frac{1}{2}H^{1\dagger}\psi_{H^1}\lambda_B \\
& -\frac{1}{2}\bar{\lambda}_B\bar{\psi}_{H^1}H^1\Big) + i\sqrt{2}g\Big(H^{2\dagger}\frac{\tau^i}{2}\lambda^i_A\psi_{H^2} - \bar{\psi}_{H^2}\frac{\tau^i}{2}\bar{\lambda}^i_A H^2\Big) \\
& +i\sqrt{2}g'\Big(\frac{1}{2}H^{2\dagger}\psi_{H^2}\lambda_B - \frac{1}{2}\bar{\lambda}_B\bar{\psi}_{H^2}H^2\Big) + i\sqrt{2}\tilde{L}^{I\dagger}\Big(g\frac{\tau^i}{2}\lambda^i_A\psi_{L^I} \\
& -\frac{1}{2}g'\lambda_B\psi_{L^I}\Big) - i\sqrt{2}\tilde{L}^I\Big(g\frac{\tau^i}{2}\bar{\lambda}^i_A\bar{\psi}_{L^I} - \frac{1}{2}g'\bar{\lambda}_B\bar{\psi}_{L^I}\Big) \\
& +i\sqrt{2}g'\tilde{R}^{I\dagger}\lambda_B\psi_{R^I} - i\sqrt{2}g'\tilde{R}^I\bar{\lambda}_B\bar{\psi}_{R^I} - \frac{1}{2}l_I\varepsilon_{ij}\Big(\psi^i_{H^1}\psi^j_{L^I}\tilde{R}^I \\
& +\psi^i_{H^1}\psi_{R^I}\tilde{R}^I_j + \psi_{R^I}\psi^j_{L^I}H^1_i + h.c.\Big)
\end{aligned} \tag{56}$$

Now we sketch the derivation for the vertices, such as $S\tilde{\kappa}^0_i\tilde{\kappa}^0_j$ etc. Starting with the Eq. (56), we convert the pieces from two-component notations into four-component notations first, then using the spinor fields defined by Eq. (36), Eq. (37), Eq. (38), Eq. (39) and Eq. (44), we find:

$$\begin{aligned}
\mathcal{L}_{S\kappa\kappa} = & \frac{\sqrt{g^2+g'^2}}{2}\Big[C^{ij}_{snn}H^0_i\bar{\kappa}^0_j P_L\kappa^0_m + C^{ij*}_{snn}H^0_i\bar{\kappa}^0_j P_R\kappa^0_m\Big] \\
& +\frac{g}{\sqrt{2}}\Big[C^{ij}_{skk}H^0_i\bar{\kappa}^+_m P_L\kappa^+_j + C^{ij*}_{skk}H^0_i\bar{\kappa}^+_j P_R\kappa^+_m\Big]
\end{aligned}$$

$$
\begin{aligned}
&+i\frac{\sqrt{g^2+g'^2}}{2}\Big[C_{onn}^{ij}H_{5+i}^0\bar{\kappa}_j^0 P_R\kappa_m^0 - C_{onn}^{ij*}H_{5+i}^0\bar{\kappa}_m^0 P_L\kappa_j^0\Big] \\
&+i\frac{g}{\sqrt{2}}\Big[C_{okk}^{ij}H_{5+i}^0\bar{\kappa}_m^+ P_L\kappa_j^+ - C_{okk}^{ij*}H_{5+i}^0\bar{\kappa}_m^+ P_R\kappa_j^+\Big] \\
&+\sqrt{g^2+g'^2}\Big[C_{Lnk}^{ij}\bar{\kappa}_j^+ P_L\kappa_m^0 H_i^+ - C_{Rnk}^{ij}\bar{\kappa}_j^+ P_R\kappa_m^0 H_i^+\Big]
\end{aligned}
\tag{57}
$$

with the definitions of C_{snn}^{ij}, C_{Lnk}^{ij}, C_{Rnk}^{ij} and C_{skk}^{ij} are given as

$$
\begin{aligned}
C_{snn}^{ij} &= \Big(\cos\theta_W Z_N^{j,2} - \sin\theta_W Z_N^{j,1}\Big)\Big(\sum_{\alpha=1}^{5} Z_{even}^{i,\alpha} Z_N^{m,2+\alpha} - 2Z_{even}^{i,2}Z_N^{m,4}\Big), \\
C_{skk}^{ij} &= \Big(Z_{even}^{i,1}Z_+^{j,1}Z_-^{m,2} + Z_{even}^{i,2}Z_+^{j,2}Z_-^{m,1} + \sum_{\alpha=3}^{5} Z_{even}^{i,\alpha}Z_+^{j,1}Z_-^{m,\alpha}\Big) \\
&\quad +\frac{1}{2g}\sum_{I=1}^{3} l_I\Big(Z_{even}^{i,I+2}Z_+^{j,I+2}Z_-^{m,2} - Z_{even}^{i,1}Z_+^{j,I+2}Z_-^{m,I+2}\Big), \\
C_{onn}^{ij} &= \Big(\cos\theta_W Z_N^{j,2} - \sin\theta_W Z_N^{j,1}\Big)\Big(\sum_{\alpha=1}^{5} Z_{odd}^{i,\alpha} Z_N^{m,2+\alpha} - 2Z_{odd}^{i,2}Z_N^{m,4}\Big), \\
C_{okk}^{ij} &= \Big(Z_{odd}^{i,1}Z_+^{j,1}Z_-^{m,2} + Z_{odd}^{i,2}Z_+^{j,2}Z_-^{m,1} + \sum_{\alpha=1}^{3} Z_{odd}^{i,2+\alpha}Z_+^{j,1}Z_-^{m,2+\alpha}\Big) \\
&\quad +\frac{i}{2g}\sum_{I} l_I\Big(Z_{odd}^{i,2+I}Z_+^{j,2+I}Z_-^{m,2} - Z_{odd}^{i,1}Z_+^{j,2+I}Z_-^{m,2+I}\Big), \\
C_{Lnk}^{ij} &= \Big[Z_c^{i,1}\Big(\frac{1}{\sqrt{2}}\Big(\cos\theta_W Z_-^{j,2}Z_N^{m,2} + \sin\theta_W Z_-^{j,2}Z_N^{m,1}\Big) - \cos\theta_W Z_-^{j,1}Z_N^{m,3}\Big) \\
&\quad + \sum_{\alpha=3}^{5} Z_c^{i,\alpha}\Big(\frac{1}{\sqrt{2}}\Big(\cos\theta_W Z_-^{j,\alpha}Z_N^{m,2} + \sin\theta_W Z_-^{j,\alpha}Z_N^{m,1}\Big) - \cos\theta_W Z_-^{j,1}Z_N^{m,2+\alpha}\Big)\Big] \\
&\quad +\frac{1}{2\sqrt{g^2+g'^2}}\sum_{I=1}^{3} l_I\Big(Z_c^{i,5+I}Z_-^{j,2+I}Z_N^{m,3} - Z_c^{i,5+I}Z_-^{j,2}Z_N^{m,4+I}\Big), \\
C_{Rnk}^{ij} &= \Big[Z_c^{i,2}\Big(\frac{1}{\sqrt{2}}\Big(\cos\theta_W Z_+^{*j,2}Z_N^{*m,2} + \sin\theta_W Z_+^{*j,2}Z_N^{*m,1}\Big) \\
&\quad + \cos\theta_W Z_+^{*j,1}Z_N^{*m,4}\Big) + \sqrt{2}\sin\theta_W\sum_{I=1}^{3} Z_c^{i,5+I}Z_+^{*j,2+I}Z_N^{*m,1}\Big] \\
&\quad +\frac{1}{2\sqrt{g^2+g'^2}}\sum_{I=1}^{3} l_I\Big(Z_c^{i,2+I}Z_N^{*m,3}Z_+^{*j,2+I} - Z_c^{i,1}Z_N^{*m,3}Z_+^{*j,2+I}\Big).
\end{aligned}
\tag{58}
$$

Here the project operators $P_{L,R} = \frac{1\pm\gamma_5}{2}$ and the transformation matrices $Z_\pm$, Z_N defined in Sect.II. The corresponding Feynman rules are summarized in Fig. 7. As for κ_i^0 being a Majorana fermion, we note the useful identity:

$$
\bar{\kappa}_j^0(1\pm\gamma_5)\kappa_k^0 = \bar{\kappa}_k^0(1\pm\gamma_5)\kappa_j^0,
\tag{59}
$$

which holds for anticommuting four-component Majorana spinors. This implies that the $H_i^0 \bar{\kappa}_j^0 \kappa_k^0$ interactions can be rearranged in symmetry under the interchange of j and k.

Since ν_e (e), ν_μ (μ) and ν_τ (τ) should be identified with the three lightest neutralinos (charginos) in the model, there must be some interesting phenomena relevant to them, such as κ_i^0 $(i = 1, 2, 3, 4) \to \tau H_j^+$ $(j = 2,3, \cdots, 8)$, κ_i^0 $(i = 1, 2, 3, 4) \to \nu_{e,\mu,\tau} H_j^0$ $(j = 1,\ 2,\ \cdots,$ $5)$ etc if the masses are suitable. Namely, these interactions without R-parity conservation may induce interesting rare processes [20].

D. The couplings of Gauge bosons to charginos (charged leptons) and neutralinos (neutrinos)

In this subsection we will focus on the couplings of the gauge bosons $(W,\ Z,\ \gamma)$ to the charginos (charged leptons) and neutralinos (neutrinos). Since we identify the three type charged leptons (three type neutrinos) with the three lightest charginos (neutralinos), the restrictions relating to them from the present experiments must be considered carefully. The relevant interactions come from the following pieces:

$$\begin{aligned}\mathcal{L}_{int}^{gcn} = & -i\bar{\lambda}_A^i \bar{\sigma}^\mu \mathcal{D}_\mu \lambda_A^i - i\bar{\lambda}_B \bar{\sigma}^\mu \mathcal{D}_\mu \lambda_B - i\bar{\psi}_{H^1} \bar{\sigma}^\mu \mathcal{D}_\mu \psi_{H^1} \\ & -i\bar{\psi}_{H^2} \bar{\sigma}^\mu \mathcal{D}_\mu \psi_{H^2} - i\bar{\psi}_{L^I} \bar{\sigma}^\mu \mathcal{D}_\mu \psi_{L^I} - i\bar{\psi}_{R^I} \bar{\sigma}^\mu \mathcal{D}_\mu \psi_{R^I} \end{aligned} \tag{60}$$

with

$$\begin{aligned}\mathcal{D}_\mu \lambda_A^1 &= \partial_\mu \lambda_A^1 - g A_\mu^2 \lambda_A^3 + g A_\mu^3 \lambda_A^2, \\ \mathcal{D}_\mu \lambda_A^2 &= \partial_\mu \lambda_A^2 - g A_\mu^3 \lambda_A^1 + g A_\mu^1 \lambda_A^3, \\ \mathcal{D}_\mu \lambda_A^3 &= \partial_\mu \lambda_A^3 - g A_\mu^1 \lambda_A^2 + g A_\mu^2 \lambda_A^1, \\ \mathcal{D}_\mu \lambda_B &= \partial_\mu \lambda_B, \\ \mathcal{D}_\mu \psi_{H^1} &= (\partial_\mu + igA_\mu^i \frac{\tau^i}{2} - \frac{i}{2} g' B_\mu) \psi_{H^1}, \\ \mathcal{D}_\mu \psi_{H^2} &= (\partial_\mu + igA_\mu^i \frac{\tau^i}{2} + \frac{i}{2} g' B_\mu) \psi_{H^2}, \\ \mathcal{D}_\mu \psi_{L^I} &= (\partial_\mu + igA_\mu^i \frac{\tau^i}{2} - \frac{i}{2} g' B_\mu) \psi_{L^I},\end{aligned}$$

$$\mathcal{D}_\mu \psi_{R^I} = (\partial_\mu + ig' B_\mu)\psi_{R^I}. \tag{61}$$

Similar to the couplings in $\mathcal{L}_{S\kappa\kappa}$, we convert all spinors in Eq. (60) into four component ones and using Eq. (39), Eq. (44), then we obtain:

$$\begin{aligned}
\mathcal{L}_{int}^{gcn} = & \Big\{ \sqrt{g^2+g'^2}\sin\theta_W\cos\theta_W A_\mu \bar{\kappa}_i^+ \gamma \kappa_i^+ - \sqrt{g^2+g'^2} Z_\mu \bar{\kappa}_i^+ \Big[\cos^2\theta_W \delta_{ij}\gamma^\mu \\
& + \frac{1}{2}\Big(Z_-^{*i,2} Z_-^{j,2} + \sum_{I=1}^{3} Z_-^{*i,2+I} Z_-^{j,2+I}\Big)\gamma^\mu P_R \\
& + \Big(\frac{1}{2} Z_+^{*i,2} Z_+^{j,2} - \sum_{I=1}^{3} Z_+^{*i,2+I} Z_+^{j,2+I}\Big)\gamma^\mu P_L \Big] \kappa_j^+ \Big\} \\
& + \Big\{ g\bar{\kappa}_j^+ \Big[\Big(- Z_+^{*i,1} Z_N^{j,2} + \frac{1}{\sqrt{2}} Z_+^{*i,2} Z_N^{j,4}\Big)\gamma^\mu P_L + \Big(Z_N^{*i,2} Z_-^{j,1} \\
& + \frac{1}{\sqrt{2}}\Big(Z_N^{*i,3} Z_-^{j,2} + \sum_{I=1}^{3} Z_N^{*i,4+I} Z_-^{j,2+I}\Big)\Big)\gamma^\mu P_R \Big] \kappa_i^0 W_\mu^+ + h.c. \Big\} \\
& + \frac{\sqrt{g^2+g'^2}}{2} \bar{\kappa}_i^0 \gamma^\mu \Big[\frac{1}{2}\Big(Z_N^{*i,4} Z_N^{j,4} - \Big(Z_N^{*i,3} Z_N^{j,3} + \sum_{\alpha=5}^{7} Z_N^{*i,\alpha} Z_N^{j,\alpha}\Big)\Big) P_L \\
& - \frac{1}{2}\Big(Z_N^{*i,4} Z_N^{j,4} - \Big(Z_N^{*i,3} Z_N^{j,3} + \sum_{\alpha=5}^{7} Z_N^{*i,\alpha} Z_N^{j,\alpha}\Big)\Big) P_R \Big] \kappa_j^0 Z_\mu.
\end{aligned} \tag{62}$$

The corresponding Feynman rules are summarized in Fig. 8. For we identify three lightest neutralinos (charginos) with three type neutrinos (charged leptons), we want to emphasize some features about Eq. (62):

- For the γ boson-κ-κ vertices, there is not the lepton flavor changing current interaction at the tree level, that is same as the SM and MSSM with R-parity.
- For the tree level Z boson-κ-κ vertices, there are the lepton flavor changing current interactions, this point is different from the MSSM with R-parity.
- Similar to the Z boson-κ-κ vertices, there are the tree level vertices such as $W\tau\nu_e$ which are forbidden in the MSSM with R-parity.

E. The interactions of quarks and/or squarks with charginos (charged leptons) and/or neutralinos (neutrinos)

In this subsection, we will give the Feynman rules for the interactions of quarks and squarks with charginos (charged leptons) and neutralinos (neutrinos) i.e. the $\tilde{Q}q\kappa_i^{\pm}$ vertices. Because of lepton number violation so having the mixing of neutrinos (charged leptons) and original neutralinos (charginos), the vertices may lead to interesting phenomenology, thus it is interesting to write them out. There are two pieces contributing to the above vertices. The first is the supersymmetric analogue of the $q\bar{q}W^{\pm}$ and $q\bar{q}Z$ interaction. The second is the supersymmetric analogue of the $q\bar{q}H$ interaction, which is proportional to quark mass and depends on the properties of the Higgs bosons in the model. These two kinds of vertices correspond to the terms in Eq. (31).

To consider the $\bar{q}q\kappa_i^{\pm}$ interaction first, let us write down the interaction in two-component spinors for fermions as the follows:

$$\begin{aligned}
\mathcal{L}_{\tilde{Q}q\kappa^{\pm}} = & ig\Big(C^{IJ}\tilde{Q}_2^{I*}\lambda_A^-\psi_{Q_1}^J + C^{IJ*}\tilde{Q}_1^{J*}\lambda_A^+\psi_{Q_2}^I\Big) \\
& -ig\Big(C^{IJ*}\tilde{Q}_2^I\bar{\lambda}_A^-\bar{\psi}_{Q_1}^J + C^{IJ}\tilde{Q}_1^J\bar{\lambda}_A^+\bar{\psi}_{Q_2}^I\Big) \\
& -\frac{d^I}{2}\Big(C^{IJ}\psi_{H^1}^2\psi_{Q^1}^J\tilde{D}^I + C^{IJ}\psi_{H^1}^2\tilde{Q}_1^J\psi_D^I + h.c.\Big) \\
& +\frac{u^I}{2}\Big(C^{JI*}\psi_{H^2}^1\psi_{Q^2}^J\tilde{U}^I + C^{JI*}\psi_{H^2}^1\psi_U^I\tilde{Q}_2^J + h.c.\Big),
\end{aligned} \tag{63}$$

then convert the two-component spinors into four-component spinors as discussed above:

$$\begin{aligned}
\mathcal{L}_{\tilde{Q}q\kappa^{\pm}} = & C^{IJ}\bar{\kappa}_j^+\Big[(-gZ_{D_I}^{i,1}Z_-^{j,1} + \frac{d^I}{2}Z_{D_I}^{i,2}Z_-^{j,2})P_L + \frac{u^J}{2}Z_+^{j,2*}Z_{D_I}^{i,1}P_R\Big]\psi_{u^I}\tilde{D}_{I,i}^+ \\
& +C^{IJ*}\bar{\kappa}_j^-\Big[\Big(-gZ_{U_J}^{i,1}Z_+^{j,1} + \frac{u^J}{2}Z_{U_J}^{i,2}Z_+^{j,2}\Big)P_L \\
& -\frac{d^I}{2}Z_{U_J}^{j,1*}Z_-^{j,2*}P_R\Big]\psi_{d^J}\tilde{U}_{Ji}^- + h.c.
\end{aligned} \tag{64}$$

Now ψ_{u^I}, ψ_{d^I} are four-component quark spinors of the I-th generation. The $\kappa_j^- = C\bar{\kappa}_j^{+T}$ (C is the charge-conjugation matrix) is a charged-conjugate state of κ_j^+, and κ_j^+ is defined in Eq. (44). The Feynman rules are summarized in Fig. 9.

For the $\tilde{Q}q\kappa_i^0$ interactions, we can write the pieces in two-component notations as:

$$\mathcal{L}_{\tilde{Q}q\kappa_i^0} = i\sqrt{2}\tilde{Q}^{I*}\Big(g\frac{\tau^3}{2}\lambda_A^3 + \frac{1}{6}g'\lambda_B\Big)\psi_Q^I - i\sqrt{2}\tilde{Q}^I\Big(g\frac{\tau^3}{2}\bar{\lambda}_A^3 + \frac{1}{6}g'\bar{\lambda}_B\Big)\bar{\psi}_Q^I$$
$$-i\frac{2\sqrt{2}}{3}g'\tilde{U}^{I*}\lambda_B\psi_U^I + i\frac{2\sqrt{2}}{3}g'\tilde{U}^I\bar{\lambda}_B\bar{\psi}_U^I + i\frac{\sqrt{2}}{3}g'\tilde{D}^{I*}\lambda_B\psi_D^I$$
$$-i\frac{\sqrt{2}}{3}g'\tilde{D}^I\bar{\lambda}_B\bar{\psi}_D^I + \frac{d^I}{2}\Big[\psi_{H^1}^1\psi_{Q^2}^I\tilde{D}^I + \psi_{H^1}^1\psi_D^I\tilde{Q}_2^I + h.c.\Big]$$
$$-\frac{u^I}{2}\Big[\psi_{H^2}^2\psi_{Q^1}^I\tilde{U}^I + \psi_{H^2}^2\psi_U^I\tilde{Q}_1^I + h.c.\Big] \tag{65}$$

After converting Eq. (65) into four-component notations straightforwardly and using the definition for neutralino mass eigenstates, we have:

$$\mathcal{L}_{\tilde{Q}q\kappa_i^0} = \kappa_j^0\Big\{\Big[\frac{e}{\sqrt{2}\sin\theta_W\cos\theta_W}Z_{U^I}^{i,1*}\Big(\cos\theta_W Z_N^{i,2} + \frac{1}{3}\sin\theta_W Z_N^{j,1}\Big)$$
$$-\frac{u^I}{2}Z_{U^I}^{i,1*}Z_N^{j,4*}\Big]P_L + \Big[\frac{2\sqrt{2}}{3}g'Z_{U^I}^{i,2*}Z_N^{j,1} - \frac{u^I}{2}Z_{U^I}^{i,1*}Z_N^{j,4*}\Big]P_R\Big\}\psi_{u^I}\tilde{U}_{I,i}^-$$
$$+\bar{\kappa}_j^0\Big\{\Big[\frac{e}{\sqrt{2}\sin\theta_W\cos\theta_W}Z_{D^I}^{i,1}\Big(-\cos\theta_W Z_N^{i,2} + \frac{1}{3}\sin\theta_W Z_N^{j,1}\Big)$$
$$+\frac{d^I}{2}Z_{D^I}^{i,2}Z_N^{j,3}\Big]P_L + \Big[-\frac{\sqrt{2}}{3}g'Z_{D^I}^{i,2}Z_N^{j,1}$$
$$+\frac{d^I}{2}Z_{D^I}^{i,1*}Z_N^{j,3*}\Big]P_R\Big\}\psi_{d^I}\tilde{D}_{I,i}^+ + h.c., \tag{66}$$

Thus the Feynman rules for the concerned interactions may be depicted exactly as the last two diagrams in Fig. 9.

IV. NUMERICAL RESULTS

In this section, we will analyze the mass spectrum of neutral Higgs, neutralinos and charginos numerically. We have obtained the mass matrices by set the three type sneutrinos with non-zero vacuum and $\epsilon_i \neq 0$ ($i = 1, 2, 3$). However, the matrices are too big to get the typical features. From now on, we will assume $\epsilon_1 = \epsilon_2 = 0$ and $\upsilon_{\tilde{\nu}_e} = \upsilon_{\tilde{\nu}_\mu} = 0$. i.e. only τ-lepton number is violated. We have two reasons to make the assumption:

- Under the assumption, we believe the key features will not be lost but the mass matrices will turn much simple .

- According to experimental indications, the τ-neutrino may be the heaviest among the three type neutrinos.

In the numerical calculations below, the input parameters are chosen as: $\alpha = \frac{e^2}{4\pi} = \frac{1}{128}$, $m_Z = 91.19\text{GeV}$, $m_W = 80.23\text{GeV}$, $m_\tau = 1.77\text{GeV}$, for the unknown parameters m_1, m_2, we assume $m_1 = m_2 = 1000\text{GeV}$ and the upper limit on τ-neutrino mass $m_{\nu_\tau} \leq 20\text{MeV}$ is also taken into account seriously. Now let us consider the mass matrix of neutralinos first. When $\epsilon_1 = \epsilon_2 = 0$ and $\upsilon_{\tilde{\nu}_e} = \upsilon_{\tilde{\nu}_\mu} = 0$, the Eq. (33) is simplified as:

$$\mathcal{M}_N = \begin{pmatrix} 2m_1 & 0 & -\frac{1}{2}g'\upsilon_1 & \frac{1}{2}g'\upsilon_2 & -\frac{1}{2}g'\upsilon_{\tilde{\nu}_\tau} \\ 0 & 2m_2 & \frac{1}{2}g\upsilon_1 & -\frac{1}{2}g\upsilon_2 & \frac{1}{2}g\upsilon_{\tilde{\nu}_\tau} \\ -\frac{1}{2}g'\upsilon_1 & \frac{1}{2}g\upsilon_1 & 0 & -\frac{1}{2}\mu & 0 \\ \frac{1}{2}g'\upsilon_2 & -\frac{1}{2}g\upsilon_2 & -\frac{1}{2}\mu & 0 & \frac{1}{2}\epsilon_3 \\ -\frac{1}{2}g'\upsilon_{\tilde{\nu}_\tau} & \frac{1}{2}g\upsilon_{\tilde{\nu}_\tau} & 0 & \frac{1}{2}\epsilon_3 & 0 \end{pmatrix} \tag{67}$$

As stated above, a strong restriction imposes on the matrix is from $m_{\nu_\tau} \leq 20\text{MeV}$. Ref [21] has discussed this limit impacting on the parameter space, the numerical result indicates the large value of ϵ_3 cannot be ruled out. In order to show the problem precisely, let us consider the equation for the eigenvalues of Eq. (67):

$$\begin{aligned} Det(\lambda - \mathcal{M}_N) = & \lambda^5 - 2(m_1 + m_2)\lambda^4 + \Big(4m_1m_2 - \frac{1}{4}(\epsilon_3^2 + \mu^2) - M_Z^2\Big)\lambda^3 \\ & + \Big[\frac{1}{2}(m_1 + m_2)(\epsilon_3^2 + \mu^2) + 2(m_1 + m_2)M_Z^2 \\ & + \frac{1}{4}(g^2 + g'^2)\upsilon_2(-\mu\upsilon_1 + \epsilon_3\upsilon_{\tilde{\nu}_\tau})\Big]\lambda^2 \\ & + \Big[- m_1m_2(\mu^2 + \epsilon_3^2) + \frac{1}{16}(g^2 + g'^2)(\epsilon_3\upsilon_1 + \mu\upsilon_{\tilde{\nu}_\tau})^2 \\ & + \frac{1}{2}(g^2m_1 + g'^2m_2)(\mu\upsilon_1\upsilon_2 - \epsilon_3\upsilon_2\upsilon_{\tilde{\nu}_\tau})\Big]\lambda \\ & - \frac{1}{8}(g^2m_1 + g'^2m_2)\Big(\mu\upsilon_{\tilde{\nu}_\tau} + \epsilon_3\upsilon_1\Big)^2 \\ = & \lambda^5 + \mathcal{A}_N\lambda^4 + \mathcal{B}_N\lambda^3 + \mathcal{C}_N\lambda^2 + \mathcal{D}_N\lambda + \mathcal{E}_N. \end{aligned} \tag{68}$$

For further discussions, let us introduce new symbols X, Y as:

$$X = \epsilon_3 \cos\theta_\upsilon + \mu \sin\theta_\upsilon,$$

$$Y = -\epsilon_3 \sin\theta_\upsilon + \mu \cos\theta_\upsilon, \tag{69}$$

thus we have the coefficients of Eq. (68):

$$\begin{aligned}
\mathcal{A}_N &= -2(m_1 + m_2), \\
\mathcal{B}_N &= -\frac{1}{4}(X^2 + Y^2) + 4m_1 m_2 - M_Z^2, \\
\mathcal{C}_N &= \frac{1}{2}(m_1 + m_2)(X^2 + Y^2) + 2(m_1 + m_2)M_Z^2 - M_Z^2 \cos\beta \sin\beta Y, \\
\mathcal{D}_N &= -m_1 m_2 (X^2 + Y^2) + \frac{1}{4} M_Z^2 \cos^2\beta X^2 + \frac{1}{2}(g^2 m_1 + g'^2)\upsilon^2 \sin\beta \cos\beta Y, \\
\mathcal{E}_N &= -\frac{1}{8}(g^2 m_1 + g'^2 m_2)\upsilon^2 \cos^2\beta X^2
\end{aligned} \tag{70}$$

If we fix the τ-neutrino mass m_{ν_τ} as an input parameter, so the equation $Det(\lambda - \mathcal{M}_N) = 0$ can be written as:

$$\Big(\lambda - m_{\nu_e}\Big)\Big(\lambda^4 + \mathcal{A}'_N \lambda^3 + \mathcal{B}'_N \lambda^2 + \mathcal{C}'_N \lambda + \mathcal{D}'_N\Big) = 0. \tag{71}$$

The coefficients $\mathcal{A}'_N$, $\mathcal{B}'_N$, $\mathcal{C}'_N$, $\mathcal{D}'_N$ are related to the "original" ones $\mathcal{A}_N$, $\mathcal{B}_N$, $\mathcal{C}_N$, $\mathcal{D}_N$, and $\mathcal{E}_N$ as:

$$\begin{aligned}
\mathcal{A}'_N &= \mathcal{A}_N + m_{\nu_\tau}, \\
\mathcal{B}'_N &= \mathcal{B}_N + m_{\nu_\tau}\mathcal{A}'_N, \\
\mathcal{C}'_N &= \mathcal{C}_N + m_{\nu_\tau}\mathcal{B}'_N, \\
\mathcal{D}'_N &= \mathcal{D}_N + m_{\nu_\tau}\mathcal{C}'_N = -\frac{\mathcal{E}_N}{m_{\nu_\tau}}.
\end{aligned} \tag{72}$$

To obtain the masses of the other four neutralinos, let us solve the Eq. (71) by the numerical method. In Fig. 10, we plot the mass of the lightest neutralino versus X. The three lines correspond to $m_{\nu_\tau} = 20$MeV, 2MeV and 0.2MeV respectively. From the figure, we find that the curve corresponding to $m_{\nu_\tau} = 20$MeV is the lowest and the second low one is correspond to $m_{\nu_\tau} = 2$MeV, so the tendency is that the curves are going "up" as the τ-neutrino mass is decreasing. If the mass of the lightest neutralino is not too heavy (such as $m_{\kappa_0^1} \leq 300$GeV), the absolute value of X cannot take very large (for example $|X| \leq 800$GeV).

As for the mass of the charginos, when $\epsilon_1 = \epsilon_2 = 0$, and $\upsilon_{\tilde{\nu}_e} = \upsilon_{\tilde{\nu}_\mu} = 0$, the Eq. (41) becomes:

$$\mathcal{M}_C = \begin{pmatrix} 2m_2 & \frac{e\upsilon_2}{\sqrt{2}S_W} & 0 \\ \frac{e\upsilon_1}{\sqrt{2}S_W} & \mu & \frac{l_3\upsilon_{\tilde{\nu}_\tau}}{\sqrt{2}} \\ \frac{e\upsilon_{\tilde{\nu}_\tau}}{\sqrt{2}S_W} & \epsilon_3 & \frac{l_3\upsilon_1}{\sqrt{2}} \end{pmatrix} \tag{73}$$

Because m_τ^2 should be the lightest eigenvalue of the matrix $\mathcal{M}_C^\dagger \mathcal{M}_C$, after taking the eigenvalue m_τ^2 away, the surviving eigenvalue equation becomes:

$$\lambda^2 - \mathcal{A}_c\lambda + \mathcal{B}_C = 0. \tag{74}$$

Here,

$$\begin{aligned} \mathcal{A}_C &= X^2 + Y^2 + 4m_2^2 + l_3^2\frac{\upsilon_1^2 + \upsilon_{\tilde{\nu}_\tau}^2}{2} + \frac{e^2\upsilon^2}{2S_W^2} \\ \mathcal{B}_C &= \frac{2l_3^2}{m_\tau^2}\Big\{m_2\upsilon\cos\beta Y + \frac{e^2}{4S_W^2}\upsilon^3\cos^2\beta\sin\beta\Big(\sin^2\theta_\upsilon - \cos^2\theta_\upsilon\Big)\Big\}^2 \end{aligned} \tag{75}$$

with the parameters X, Y are defined by Eq. (69). Therefore the masses of the other two charginos are expressed as:

$$m_{\kappa_{1,2}^\pm}^2 = \frac{1}{2}\Big\{\mathcal{A}_C \mp \sqrt{\mathcal{A}_C^2 - 4\mathcal{B}_C}\Big\}. \tag{76}$$

The parameter l_3 can be fixed by the condition $Det|m_\tau^2 - \mathcal{M}_c^\dagger\mathcal{M}_c| = 0$. In Fig. 11, we plot the mass of the lightest chargino versus X. The three lines correspond to $m_{\nu_\tau} = 20$MeV, 2MeV and 0.2MeV respectively. Similar to the neutralinos, we find that the curve corresponding to $m_{\nu_\tau} = 20$MeV is the lowest, the second low one is correspond to $m_{\nu_\tau} = 2$MeV and the tendency is very similar to the case for neutralinos. This can be understood as following: when the values of m_1, m_2, $\tan\beta$, $\tan\theta_\upsilon$ and X are fixed, the value of Y will be fixed by the mass of τ-neutrino. In the numerical computation, we find that the absolute value of Y turns small, as the m_{ν_τ} changes large. This is the reason why the curve corresponding to $m_{\nu_\tau} = 20$MeV is the lowest among the three curves which we have computed here.

Now, we turn to discuss the mass matrix of the neutral Higgs. Under the same assumption, the mass matrix for CP-even Higgs reduces to:

$$\mathcal{M}^2_{even} = \begin{pmatrix} r_{11} & -\frac{g^2+g'^2}{4}\upsilon_1\upsilon_2 - B\mu & \frac{g^2+g'^2}{4}\upsilon_1\upsilon_{\tilde{\nu}_\tau} - \mu\epsilon_3 \\ -\frac{g^2+g'^2}{4}\upsilon_1\upsilon_2 - B\mu & r_{22} & -\frac{g^2+g'^2}{4}\upsilon_2\upsilon_{\tilde{\nu}_\tau} + B_3\epsilon_3 \\ \frac{g^2+g'^2}{4}\upsilon_1\upsilon_{\tilde{\nu}_\tau} - \mu\epsilon_3 & -\frac{g^2+g'^2}{4}\upsilon_2\upsilon_{\tilde{\nu}_\tau} + B_3\epsilon_3 & r_{33} \end{pmatrix} \tag{77}$$

with

$$\begin{aligned} r_{11} &= \frac{g^2+g'^2}{8}(3\upsilon_1^2 - \upsilon_2^2 + \upsilon_{\tilde{\nu}_\tau}^2) + |\mu|^2 + m_{H^1}^2 \\ &= \frac{g^2+g'^2}{4}\upsilon_1^2 + \mu\epsilon_3\frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_1} + B\mu\frac{\upsilon_2}{\upsilon_1} \\ r_{22} &= \frac{g^2+g'^2}{8}(-\upsilon_1^2 + 3\upsilon_2^2 - \upsilon_{\tilde{\nu}_\tau}^2) + |\mu|^2 + |\epsilon_3|^2 + m_{H^2}^2 \\ &= \frac{g^2+g'^2}{4}\upsilon_2^2 + B\mu\frac{\upsilon_1}{\upsilon_2} - B_3\epsilon_3\frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_2} \\ r_{33} &= \frac{g^2+g'^2}{8}(\upsilon_1^2 - \upsilon_2^2 + 3\upsilon_{\tilde{\nu}_\tau}^2) + |\epsilon_3|^2 + m_{L^3}^2 \\ &= \frac{g^2+g'^2}{4}\upsilon_{\tilde{\nu}_\tau}^2 + \mu\epsilon_3\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_\tau}} - B_3\epsilon_3\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_\tau}} \end{aligned} \tag{78}$$

The mass matrix of CP-odd Higgs reduces to:

$$\mathcal{M}^2_{odd} = \begin{pmatrix} s_{11} & B\mu & -\mu\epsilon_3 \\ B\mu & s_{22} & -B_3\epsilon_3 \\ -\mu\epsilon_3 & -B_3\epsilon_3 & s_{33} \end{pmatrix} \tag{79}$$

with

$$\begin{aligned} s_{11} &= \frac{g^2+g'^2}{8}(\upsilon_1^2 - \upsilon_2^2 + \upsilon_{\tilde{\nu}_\tau}^2) + |\mu|^2 + m_{H^1}^2, \\ &= \mu\epsilon_3\frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_1} + B\mu\frac{\upsilon_2}{\upsilon_1} \\ s_{22} &= -\frac{g^2+g'^2}{8}(\upsilon_1^2 - \upsilon_2^2 + \upsilon_{\tilde{\nu}_\tau}^2) + |\mu|^2 + |\epsilon_3|^2 + m_{H^2}^2 \\ &= B\mu\frac{\upsilon_1}{\upsilon_2} - B_3\epsilon_3\frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_2}, \\ s_{33} &= \frac{g^2+g'^2}{8}(\upsilon_1^2 - \upsilon_2^2 + \upsilon_{\tilde{\nu}_\tau}^2) + |\epsilon_3|^2 + m_{L^3}^2 \\ &= \mu\epsilon_3\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_\tau}} - B_3\epsilon_3\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_\tau}}. \end{aligned} \tag{80}$$

Introducing the following variables:

$$X_s = B\mu,$$

$$Y_s = \mu\epsilon_3,$$

$$Z_s = B_3\epsilon_3, \tag{81}$$

the masses of the neutral Higgs can be determined from the X_s, Y_s, Z_s and $\tan\beta$, $\tan\theta_\upsilon$. For the masses of CP-odd Higgs, we define:

$$\begin{aligned}
\mathcal{A} &= X_s\Big(\frac{\upsilon_1}{\upsilon_2}+\frac{\upsilon_2}{\upsilon_1}\Big)+Y_s\Big(\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_\tau}}+\frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_1}\Big)-Z_s\Big(\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_\tau}}+\frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_2}\Big),\\
\mathcal{B} &= -Y_sZ_s\Big(\frac{\upsilon_1}{\upsilon_2}+\frac{\upsilon_2}{\upsilon_1}\Big)-X_sZ_s\Big(\frac{\upsilon_1}{\upsilon_{\tilde{\nu}_\tau}}+\frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_1}\Big)+X_sY_s\Big(\frac{\upsilon_2}{\upsilon_{\tilde{\nu}_\tau}}+\frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_2}\Big)\\
&\quad +X_sY_s\frac{\upsilon_1^2}{\upsilon_2\upsilon_{\tilde{\nu}_\tau}}-X_sZ_s\frac{\upsilon_2^2}{\upsilon_1\upsilon_{\tilde{\nu}_\tau}}-Y_sZ_s\frac{\upsilon_{\tilde{\nu}_\tau}^2}{\upsilon_1\upsilon_2},
\end{aligned} \tag{82}$$

the masses of the two CP-odd Higgs can be given as:

$$m^2_{H^0_{3+2,3}} = \frac{1}{2}\Big(\mathcal{A}\mp\sqrt{\mathcal{A}^2-4\mathcal{B}}\Big). \tag{83}$$

In Fig. 12, we plot the mass of the lightest CP-odd Higgs versus the mass of the lightest CP-even Higgs, where the ranges of the parameters are: $-10^5\text{GeV}^{\ 2} \le X_s, Y_s, Z_s \le 10^5\text{GeV}^2$ and $.5 \le \tan\beta, \tan\theta_\upsilon \le 50$. From the Fig. 12, we can find that there are no limit on the $m_{H^0_5}$ when we change those parameters in the above ranges. As for the lightest CP-even Higgs, the difference from the MSSM with R-parity is that $m_{H^0_1}$ can larger than m_Z at the tree level. This can be understood from Eq. (26), under the assumptions, we have

$$m^2_{H^0_1} \le m_{H^0_3} m_Z \cos 2\beta \frac{1-\frac{1}{2}\frac{m_Z^2}{m^2_{H^0_3}}}{1-\frac{1}{2}\Big(\frac{m_Z^2\cos^2 2\beta}{m^2_{H^0_3}}\Big)^{\frac{1}{2}}} \tag{84}$$

where the $m_{H^0_3}$ is the mass of the heaviest CP-even Higgs in this case and we cannot give the stringent limit on it as in the MSSM with R-parity.

In summary, we have analyzed the mass spectrum in the MSSM with bilinear R-parity violation. From the restriction $m_{\nu_\tau} \le 20$MeV, we cannot rule out the possibilities with large ϵ_3 and $\upsilon_{\tilde{\nu}_\tau}$. We also derived the Feynman rules in the $\prime$t-Hooft Feynman gauge, which are convenient when we study the phenomenology beyond the tree level in the model. Recent

experimental signals of neutrino masses and mixing may provide the first glimpses of the lepton number violation effects, Ref [22] have study the Neutrino Oscillations experiment constraint on the parameter space of the model. Considering both the fermionic and scalar sectors, they find that a large area of the parameter space is allowed. Here, we would also like to point out some references have analyzed the $0\nu\beta\beta$-decay in the model [23] and obtained new stringent upper limits on the first generation R-parity violating parameters, ϵ_1 and $\upsilon_{\tilde{\nu}_e}$; whereas for the other two generations, there are not very serious restrictions on the upper limits of the R-parity violating parameters. As for other interesting processes in the model, they are discussed by Ref [24].

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APPENDIX A: THE MASS MATRIX OF CHARGED HIGGS

In the case of charged Higgs, with the current basis $\Phi_c = (H_2^{1*}, H_1^2, \tilde{L}_2^{1*}, \tilde{L}_2^{2*}, \tilde{L}_2^{3*}, \tilde{R}^1, \tilde{R}^2, \tilde{R}^3)$, the symmetric matrix $\mathcal{M}_c^2$ is given as follows:

$$\begin{aligned}
\mathcal{M}^2_{c1,1} &= \frac{g^2}{4}\upsilon_1^2 - \frac{g^2-g'^2}{8}(\upsilon_1^2-\upsilon_2^2+\sum_I \upsilon_{\tilde{\nu}_I}^2) + \mu^2 + \sum_I \frac{1}{2} l_I^2 \upsilon_{\tilde{\nu}_I}^2 + m^2_{H^1} \\
&= \frac{g^2}{4}(\upsilon_2^2 - \sum_I \upsilon_{\tilde{\nu}_I}^2) + \sum_I \frac{1}{2} l_I^2 \upsilon_{\tilde{\nu}_I}^2 + \sum_I \mu\epsilon_I \frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_1} + B\mu\frac{\upsilon_2}{\upsilon_1}, \\
\mathcal{M}^2_{c1,2} &= \frac{g^2}{4}\upsilon_1\upsilon_2 + B\mu, \\
\mathcal{M}^2_{c1,3} &= \frac{g^2}{4}\upsilon_1\upsilon_{\tilde{\nu}_e} - \mu\epsilon_1 - \frac{1}{2} l_1^2 \upsilon_1 \upsilon_{\tilde{\nu}_e}, \\
\mathcal{M}^2_{c1,4} &= \frac{g^2}{4}\upsilon_1\upsilon_{\tilde{\nu}_\mu} - \mu\epsilon_2 - \frac{1}{2} l_2^2 \upsilon_1 \upsilon_{\tilde{\nu}_\mu}, \\
\mathcal{M}^2_{c1,5} &= \frac{g^2}{4}\upsilon_1\upsilon_{\tilde{\nu}_\tau} - \mu\epsilon_3 - \frac{1}{2} l_3^2 \upsilon_1 \upsilon_{\tilde{\nu}_\tau}, \\
\mathcal{M}^2_{c1,6} &= \frac{1}{\sqrt{2}} l_1 \epsilon_1 \upsilon_2 + l_{s_1}\frac{\mu\upsilon_{\tilde{\nu}_e}}{\sqrt{2}}, \\
\mathcal{M}^2_{c1,7} &= \frac{1}{\sqrt{2}} l_2 \epsilon_2 \upsilon_2 + l_{s_2}\frac{\mu\upsilon_{\tilde{\nu}_\mu}}{\sqrt{2}},
\end{aligned}$$

$$
\begin{aligned}
\mathcal{M}^2_{c1,8} &= \frac{1}{\sqrt{2}} l_3 \epsilon_3 \upsilon_2 + l_{s_3} \frac{\mu \upsilon_{\tilde{\nu}_\tau}}{\sqrt{2}}, \\
\mathcal{M}^2_{c2,2} &= \frac{g^2}{4}\upsilon_2^2 + \frac{1}{8}(g^2 - g'^2)(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2) + \mu^2 + \sum_I \epsilon_I^2 + m^2_{H^2} \\
&= \frac{g^2}{4}(\upsilon_1^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2) - \sum_I B_I \epsilon_I \frac{\upsilon_{\tilde{\nu}_I}}{\upsilon_2} + B\mu \frac{\upsilon_1}{\upsilon_2}, \\
\mathcal{M}^2_{c2,3} &= \frac{g^2}{4}\upsilon_2 \upsilon_{\tilde{\nu}_e} - B_1 \epsilon_1, \\
\mathcal{M}^2_{c2,4} &= \frac{g^2}{4}\upsilon_2 \upsilon_{\tilde{\nu}_\mu} - B_2 \epsilon_2, \\
\mathcal{M}^2_{c2,5} &= \frac{g^2}{4}\upsilon_2 \upsilon_{\tilde{\nu}_\tau} - B_3 \epsilon_3, \\
\mathcal{M}^2_{c2,6} &= \frac{l_1}{\sqrt{2}}\mu \upsilon_{\tilde{\nu}_e} + \frac{l_1}{\sqrt{2}}\epsilon_1 \upsilon_1, \\
\mathcal{M}^2_{c2,7} &= \frac{l_2}{\sqrt{2}}\mu \upsilon_{\tilde{\nu}_\mu} + \frac{l_2}{\sqrt{2}}\epsilon_2 \upsilon_1, \\
\mathcal{M}^2_{c2,8} &= \frac{l_3}{\sqrt{2}}\mu \upsilon_{\tilde{\nu}_\tau} + \frac{l_3}{\sqrt{2}}\epsilon_3 \upsilon_1, \\
\mathcal{M}^2_{c3,3} &= \frac{g^2}{4}\upsilon_{\tilde{\nu}_e}^2 - \frac{1}{8}(g^2 - g'^2)(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2) + \epsilon_1^2 + \frac{l_1^2}{2}\upsilon_1^2 + m^2_{L^1} \\
&= \frac{g^2}{4}(\upsilon_2^2 - \upsilon_1^2) + \epsilon_1 \frac{\mu \upsilon_1}{\upsilon_{\tilde{\nu}_e}} - B_1 \frac{\epsilon_1 \upsilon_2}{\upsilon_{\tilde{\nu}_e}} + \frac{l_1^2}{2}\upsilon_1^2 \\
&\quad -\epsilon_1 \epsilon_2 \frac{\upsilon_{\tilde{\nu}_\mu}}{\upsilon_{\tilde{\nu}_e}} - \epsilon_1 \epsilon_3 \frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_{\tilde{\nu}_e}} - \frac{g^2}{4}(\upsilon_{\tilde{\nu}_\mu}^2 + \upsilon_{\tilde{\nu}_\tau}^2), \\
\mathcal{M}^2_{c3,4} &= \frac{g^2}{4}\upsilon_{\tilde{\nu}_e}\upsilon_{\tilde{\nu}_\mu} + \epsilon_1 \epsilon_2, \\
\mathcal{M}^2_{c3,5} &= \frac{g^2}{4}\upsilon_{\tilde{\nu}_e}\upsilon_{\tilde{\nu}_\tau} + \epsilon_1 \epsilon_3, \\
\mathcal{M}^2_{c3,6} &= \frac{1}{\sqrt{2}} l_1 \mu \upsilon_2 + \frac{1}{\sqrt{2}} l_{s_1} \mu \upsilon_1, \\
\mathcal{M}^2_{c3,7} &= 0, \\
\mathcal{M}^2_{c3,8} &= 0, \\
\mathcal{M}^2_{c4,4} &= \frac{g^2}{4}\upsilon_{\tilde{\nu}_\mu}^2 - \frac{1}{8}(g^2 - g'^2)(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon_{\tilde{\nu}_I}^2) + \epsilon_2^2 + \frac{l_2^2}{2}\upsilon_1^2 + m^2_{L^2} \\
&= \frac{g^2}{4}(\upsilon_2^2 - \upsilon_1^2) + \epsilon_2 \frac{\mu \upsilon_1}{\upsilon_{\tilde{\nu}_\mu}} - B_2 \frac{\epsilon_2 \upsilon_2}{\upsilon_{\tilde{\nu}_\mu}} + \frac{l_2^2}{2}\upsilon_1^2 \\
&\quad -\epsilon_1 \epsilon_2 \frac{\upsilon_{\tilde{\nu}_e}}{\upsilon_{\tilde{\nu}_\mu}} - \epsilon_2 \epsilon_3 \frac{\upsilon_{\tilde{\nu}_\tau}}{\upsilon_{\tilde{\nu}_\mu}} - \frac{g^2}{4}(\upsilon_{\tilde{\nu}_e}^2 + \upsilon_{\tilde{\nu}_\tau}^2), \\
\mathcal{M}^2_{c4,5} &= \frac{g^2}{4}\upsilon_{\tilde{\nu}_\mu}\upsilon_{\tilde{\nu}_\tau},
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}^2_{c4,6} &= 0, \\
\mathcal{M}^2_{c4,7} &= \frac{1}{\sqrt{2}} l_2 \mu \upsilon_2 - \frac{1}{\sqrt{2}} l_{s_2} \mu \upsilon_1, \\
\mathcal{M}^2_{c4,8} &= 0, \\
\mathcal{M}^2_{c5,5} &= \frac{g^2}{4} \upsilon^2_{\tilde{\nu}_\tau} - \frac{1}{8}(g^2 - g'^2)(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon^2_{\tilde{\nu}_I}) + \epsilon_3^2 + \frac{l_3^2}{2}\upsilon_1^2 + m^2_{L^3} \\
&= \frac{g^2}{4}(\upsilon_2^2 - \upsilon_1^2) + \epsilon_3 \frac{\mu \upsilon_1}{\upsilon_{\tilde{\nu}_\tau}} - B_3 \frac{\epsilon_3 \upsilon_2}{\upsilon_{\tilde{\nu}_\tau}} + \frac{l_3^2}{2}\upsilon_1^2 \\
&\quad - \epsilon_1 \epsilon_3 \frac{\upsilon_{\tilde{\nu}_e}}{\upsilon_{\tilde{\nu}_\tau}} - \epsilon_2 \epsilon_3 \frac{\upsilon_{\tilde{\nu}_\mu}}{\upsilon_{\tilde{\nu}_\tau}} - \frac{g^2}{4}(\upsilon^2_{\tilde{\nu}_e} + \upsilon^2_{\tilde{\nu}_\mu}), \\
\mathcal{M}^2_{c5,6} &= 0, \\
\mathcal{M}^2_{c5,7} &= 0, \\
\mathcal{M}^2_{c5,8} &= \frac{1}{\sqrt{2}} l_3 \mu \upsilon_{\tilde{\nu}_\tau} - \frac{1}{\sqrt{2}} l_{s_3} \mu \upsilon_1, \\
\mathcal{M}^2_{c6,6} &= -\frac{g'^2}{4}(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon^2_{\tilde{\nu}_I}) + \frac{1}{2} l_1^2 (\upsilon_1^2 + \upsilon^2_{\tilde{\nu}_e}) + m^2_{R^1}, \\
\mathcal{M}^2_{c6,7} &= \frac{1}{2} l_1 l_2 \upsilon_{\tilde{\nu}_e} \upsilon_{\tilde{\nu}_\mu}, \\
\mathcal{M}^2_{c6,8} &= \frac{1}{2} l_1 l_3 \upsilon_{\tilde{\nu}_e} \upsilon_{\tilde{\nu}_\tau}, \\
\mathcal{M}^2_{c7,7} &= -\frac{g'^2}{4}(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon^2_{\tilde{\nu}_I}) + \frac{1}{2} l_1^2 (\upsilon_1^2 + \upsilon^2_{\tilde{\nu}_\mu}) + m^2_{R^2}, \\
\mathcal{M}^2_{c7,8} &= \frac{1}{2} l_2 l_3 \upsilon_{\tilde{\nu}_\mu} \upsilon_{\tilde{\nu}_\tau}, \\
\mathcal{M}^2_{c8,8} &= -\frac{g'^2}{4}(\upsilon_1^2 - \upsilon_2^2 + \sum_I \upsilon^2_{\tilde{\nu}_I}) + \frac{1}{2} l_3^2 (\upsilon_1^2 + \upsilon^2_{\tilde{\nu}_\tau}) + m^2_{R^3}.
\end{aligned}
\tag{A1}
$$

Note here that to obtain Eq. (A1), Eq. (12) is used sometimes.

APPENDIX B: THE MIXING OF THE SQUARKS

In a general case, the matrix of the squarks mixing should be 6×6. Under our assumptions, we do not consider the squarks mixing between different generations. From superpotential Eq. (2) and the soft-breaking terms, we find the up squarks mass matrix of the I-th generation can be written as:

$$\mathcal{M}^2_{U^I} = \begin{pmatrix} \frac{1}{24}(3g^2 - g'^2)(\upsilon^2 - 2\upsilon_2^2) + \frac{u_I^2}{2}\upsilon_2^2 + m^2_{Q^I} & \frac{1}{\sqrt{2}}(u_I\mu\upsilon_1 - u_I\sum\limits_{J=1}^{3}\epsilon_J\upsilon_{\tilde{\nu}_J} - u_{S_I}\mu\upsilon_2) \\ \frac{1}{\sqrt{2}}(u_I\mu\upsilon_1 - u_I\sum\limits_{J=1}^{3}\epsilon_J\upsilon_{\tilde{\nu}_J} - u_{S_I}\mu\upsilon_2) & \frac{1}{6}g'^2(\upsilon^2 - 2\upsilon_2^2) + \frac{u_I^2}{2}\upsilon_2^2 + m^2_{U^I} \end{pmatrix} \tag{B1}$$

where $I = (1,\ 2,\ 3)$ is the index of the generations. The current eigenstates $\tilde{Q}_1^I$ and $\tilde{U}^I$ connect to the two physical (mass) eigenstates $\tilde{U}_I^i(i = (1,\ 2)$ through

$$\tilde{U}_I^i = Z^{i,1}_{U^I}\tilde{Q}_1^I + Z^{i,2}_{U^I}\tilde{U}^I \tag{B2}$$

and Z_{U^I} is determined by the condition:

$$Z^{\dagger}_{U^I}\mathcal{M}^2_{U^I}Z_{U^I} = \mathrm{diag}(M^2_{U^1_I}, M^2_{U^2_I}) \tag{B3}$$

In a similar way, we can give the down squarks mass matrix of the I-th generation:

$$\mathcal{M}^2_{D^I} = \begin{pmatrix} -\frac{1}{24}(3g^2 + g'^2)(\upsilon^2 - 2\upsilon_2^2) + \frac{d_I^2}{2}\upsilon_1^2 + m^2_{Q^I} & -\frac{1}{\sqrt{2}}(d_I\mu\upsilon_2 - d_{S_I}\mu\upsilon_1) \\ -\frac{1}{\sqrt{2}}(d_I\mu\upsilon_2 - d_{S_I}\mu\upsilon_1) & -\frac{1}{12}g'^2(\upsilon^2 - 2\upsilon_2^2) + \frac{d_I^2}{2}\upsilon_1^2 + m^2_{D^I} \end{pmatrix} \tag{B4}$$

The fields $\tilde{Q}_2^I$ and $\tilde{D}^I$ relate to the two physical (mass) eigenstates $\tilde{D}_I^i$ $(i = (1,\ 2)$:

$$\begin{aligned} &\tilde{D}_I^i = Z^{i,1}_{D^I}\tilde{Q}_2^I + Z^{i,2}_{D^I}\tilde{D}^I \\ &Z^{\dagger}_{D^I}\mathcal{M}^2_{D^I}Z_{D^I} = \mathrm{diag}(M^2_{D^1_I}, M^2_{D^2_I}) \end{aligned} \tag{B5}$$

APPENDIX C: EXPRESSIONS OF THE COUPLINGS IN $\mathcal{L}_{SSS}$ AND $\mathcal{L}_{SSSS}$

In this appendix, we give precise expressions of the couplings that appear in the $\mathcal{L}_{SSS}$ and $\mathcal{L}_{SSSS}$. The method has been described clearly in text, the results are:

$$\begin{aligned} A^{kij}_{ec} = &\frac{g^2 + g'^2}{4}\Big(\upsilon_1 Z^{k,1}_{even}Z^{i,1}_c Z^{j,1}_c + \upsilon_2 Z^{k,2}_{even}Z^{i,2}_c Z^{j,2}_c + \sum_{I=1}^{3}\upsilon_{\tilde{\nu}_I}Z^{k,2+I}_{even}Z^{i,2+I}_c Z^{j,2+I}_c\Big) \\ &+ \sum_{I=1}^{3}\Big(\frac{g'^2 - g^2}{4} + l_I^2\Big)\Big(\upsilon_1 Z^{k,1}_{even}Z^{i,2+I}_c Z^{j,2+I}_c + \upsilon_{\tilde{\nu}_I}Z^{k,2+I}_{even}Z^{i,1}_c Z^{j,1}_c\Big) \\ &+ \sum_{I=1}^{3}\Big(\frac{g^2}{4} - \frac{1}{2}l_I^2\Big)\Big\{\upsilon_1 Z^{k,2+I}_{even}\Big(Z^{i,2+I}_c Z^{j,2}_c + Z^{i,2}_c Z^{j,2+I}_c\Big) + \upsilon_{\tilde{\nu}_I}Z^{k,1}_{even}\Big(Z^{i,2+I}_c Z^{j,2}_c \end{aligned}$$

$$
\begin{aligned}
&+Z_c^{i,2}Z_c^{j,2+I}\Big)\Big\}+\frac{g^2-g'^2}{4}\Big(\upsilon_1 Z_{even}^{k,1}Z_c^{i,2}Z_c^{j,2}+\upsilon_2 Z_{even}^{k,2}Z_c^{i,1}Z_c^{j,1}\\
&+\sum_{I=1}^{3}\upsilon_{\tilde{\nu}_I}Z_{even}^{k,2+I}Z_c^{i,2}Z_c^{j,2}+\upsilon_2 Z_{even}^{k,2}Z_c^{i,2+I}Z_c^{j,2+I}\Big)\\
&+\sum_{I=1}^{3}\Big[\Big(l_I^2-\frac{g'^2}{2}\Big)\upsilon_{\tilde{\nu}_I}Z_{even}^{k,2+I}Z_c^{i,5+I}Z_c^{j,5+I}+\Big(l_I^2-\frac{g'^2}{2}\Big)\upsilon_1 Z_{even}^{k,1}Z_c^{i,5+I}Z_c^{j,5+I}\\
&+\frac{g'^2}{2}\upsilon_2 Z_{even}^{k,2}Z_c^{i,5+I}Z_c^{j,5+I}\Big]+\frac{g^2}{4}\Big(\upsilon_{\tilde{\nu}_I}Z_{even}^{k,2}+\upsilon_2 Z_{even}^{k,2+I}\Big)\Big(Z_c^{i,2+I}Z_c^{j,2}\\
&+Z_c^{i,2}Z_c^{j,2+I}\Big)+\frac{g^2}{4}\Big(\upsilon_1 Z_{even}^{k,2}+\upsilon_2 Z_{even}^{k,1}\Big)\Big(Z_c^{i,1}Z_c^{j,2}\\
&+Z_c^{i,2}Z_c^{j,1}\Big)+\frac{1}{\sqrt{2}}l_I\epsilon_3 Z_{even}^{k,1}\Big(Z_c^{i,4}Z_c^{j,2}\\
&+Z_c^{i,2}Z_c^{j,4}\Big)+\frac{1}{\sqrt{2}}\sum_{I=1}^{3}l_I\epsilon_I Z_{even}^{k,2}\Big(Z_c^{i,5+I}Z_c^{j,1}+Z_c^{i,1}Z_c^{j,5+I}\Big)
\end{aligned}
$$

$$
\begin{aligned}
A_{oc}^{kij}=&\sum_{I=1}^{3}\Big\{\Big(\frac{g^2}{4}-l_I^2\Big)\Big[\upsilon_{\tilde{\nu}_I}Z_{odd}^{k,1}\Big(Z_c^{i,1}Z_c^{j,2+I}-Z_c^{i,2+I}Z_c^{j,1}\Big)\\
&+\upsilon_1 Z_{odd}^{k,2+I}\Big(Z_c^{i,1}Z_c^{j,2+I}-Z_c^{i,2+I}Z_c^{j,1}\Big)\Big]\\
&+\frac{g^2}{4}\Big(\upsilon_{\tilde{\nu}_I}Z_{odd}^{k,2}+\upsilon_2 Z_{odd}^{k,2+I}\Big)\Big(Z_c^{i,2+I}Z_c^{j,2}-Z_c^{i,2}Z_c^{j,2+I}\Big)+\frac{g^2}{4}\Big(\upsilon_{\tilde{\nu}_I}Z_{odd}^{k,2}\\
&+\upsilon_2 Z_{odd}^{k,2+I}\Big)\Big(Z_c^{i,2+I}Z_c^{j,2}-Z_c^{i,2}Z_c^{j,2+I}\Big)\\
&\frac{1}{\sqrt{2}}l_I\epsilon_I Z_{odd}^{k,1}\Big(-Z_c^{i,5+I}Z_c^{j,2}+Z_c^{i,2}Z_c^{j,5+I}\Big)\\
&-\frac{1}{\sqrt{2}}l_I\epsilon_I Z_{odd}^{k,2}\Big(Z_c^{i,5+I}Z_c^{j,1}-Z_c^{i,1}Z_c^{j,5+I}\Big)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{ec}^{klij}=&\frac{g^2+g'^2}{8}\Big(Z_{even}^{k,1}Z_{even}^{l,1}Z_c^{i,1}Z_c^{j,1}+Z_{even}^{k,2}Z_{even}^{l,2}Z_c^{i,2}Z_c^{j,2}+\sum_{I=1}^{3}Z_{even}^{k,2+I}Z_{even}^{l,2+I}Z_c^{i,2+I}Z_c^{j,2+I}\Big)\\
&+\sum_{I=1}^{3}\Big(\frac{g'^2-g^2}{8}+\frac{1}{2}l_I^2\Big)\Big(Z_{even}^{k,1}Z_{even}^{l,1}Z_c^{i,2+I}Z_c^{j,2+I}+Z_{even}^{k,2+I}Z_{even}^{l,2+I}Z_c^{i,1}Z_c^{j,1}\Big)\\
&+\frac{g'^2-g^2}{8}\Big[Z_{even}^{k,1}Z_{even}^{l,1}Z_c^{i,2}Z_c^{j,2}+Z_{even}^{k,2}Z_{even}^{l,2}Z_c^{i,1}Z_c^{j,1}\\
&+\sum_{I=1}^{3}\Big(Z_{even}^{k,2+I}Z_{even}^{l,2+I}Z_c^{i,2}Z_c^{j,2}+Z_{even}^{k,2}Z_{even}^{l,2}Z_c^{i,2+I}Z_c^{j,2+I}\Big)\Big]\\
&+\frac{g^2}{4}\Big[Z_{even}^{k,1}Z_{even}^{l,2}\Big(Z_c^{i,2}Z_c^{j,1}+Z_c^{i,1}Z_c^{j,2})+\sum_{I=1}^{3}Z_{even}^{k,1}Z_{even}^{l,2+I}(Z_c^{i,2+I}Z_c^{j,1}\\
&+Z_c^{i,1}Z_c^{j,2+I}\Big)+\sum_{I=1}^{3}Z_{even}^{k,2}Z_{even}^{l,2+I}\Big(Z_c^{i,2+I}Z_c^{j,2}+Z_c^{i,2}Z_c^{j,2+I}\Big)\Big]
\end{aligned}
$$

$$
\begin{aligned}
&-\sum_{I=1}^{3}\Big[\frac{l_I^2}{2}Z_{even}^{k,1}Z_{even}^{l,2+I}\Big(Z_c^{i,2+I}Z_c^{j,1}+Z_c^{i,1}Z_c^{j,2+I}\Big)-\frac{g'^2}{4}\Big(Z_{even}^{k,1}Z_{even}^{l,1}Z_c^{i,5+I}Z_c^{j,5+I}\\
&-Z_{even}^{k,2}Z_{even}^{l,2}Z_c^{i,5+I}Z_c^{j,5+I}+Z_{even}^{k,2+I}Z_{even}^{l,2+I}Z_c^{i,5+I}Z_c^{j,5+I}\Big)\\
&+\frac{l_I^2}{2}Z_{even}^{k,1}Z_{even}^{l,1}Z_c^{i,5+I}Z_c^{j,5+I}\Big]
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{oc}^{klij}=&\frac{g^2+g'^2}{8}\Big(Z_{odd}^{k,1}Z_{odd}^{l,1}Z_c^{i,1}Z_c^{j,1}+Z_{odd}^{k,2}Z_{odd}^{l,2}Z_c^{i,2}Z_c^{j,2}\\
&+\sum_{I=1}^{3}Z_{odd}^{k,2+I}Z_{odd}^{l,2+I}Z_c^{i,2+I}Z_c^{j,2+I}\Big)+\sum_{I=1}^{3}\Big(\frac{g'^2-g^2}{8}+\frac{1}{2}l_I^2\Big)\Big(Z_{odd}^{k,1}Z_{odd}^{l,1}Z_c^{i,2+I}Z_c^{j,2+I}\\
&+Z_{odd}^{k,2+I}Z_{odd}^{l,2+I}Z_c^{i,1}Z_c^{j,1}\Big)+\frac{g'^2-g^2}{8}\Big\{Z_{odd}^{k,1}Z_{odd}^{l,1}Z_c^{i,2}Z_c^{j,2}\\
&+Z_{odd}^{k,2}Z_{odd}^{l,2}Z_c^{i,1}Z_c^{j,1}+\sum_{I=1}^{3}\Big(Z_{odd}^{k,2+I}Z_{odd}^{l,2+I}Z_c^{i,2}Z_c^{j,2}\\
&+Z_{odd}^{k,2}Z_{odd}^{l,2}Z_c^{i,2+I}Z_c^{j,2+I}\Big)\Big\}+\frac{g^2}{4}\Big\{Z_{odd}^{k,1}Z_{odd}^{l,2}\Big(Z_c^{i,2}Z_c^{j,1}+Z_c^{i,1}Z_c^{j,2}\Big)\\
&+\sum_{I=1}^{3}\Big[Z_{odd}^{k,1}Z_{odd}^{l,3}\Big(Z_c^{i,3}Z_c^{j,1}+Z_c^{i,1}Z_c^{j,3}\Big)+Z_{odd}^{k,2}Z_{odd}^{l,3}\Big(Z_c^{i,3}Z_c^{j,2}+Z_c^{i,2}Z_c^{j,3}\Big)\Big]\Big\}\\
&-\sum_{I=1}^{3}\Big\{\frac{l_I^2}{2}Z_{odd}^{k,1}Z_{odd}^{l,2+I}\Big(Z_c^{i,2+I}Z_c^{j,1}+Z_c^{i,1}Z_c^{j,2+I}\Big)-\frac{g'^2}{4}\Big(Z_{odd}^{k,1}Z_{odd}^{l,1}Z_c^{i,5+I}Z_c^{j,5+I}\\
&-Z_{odd}^{k,2}Z_{odd}^{l,2}Z_c^{i,5+I}Z_c^{j,5+I}+Z_{odd}^{k,2+I}Z_{odd}^{l,2+I}Z_c^{i,5+I}Z_c^{j,5+I}\Big)\\
&+\frac{l_I^2}{2}Z_{odd}^{k,1}Z_{odd}^{l,1}Z_c^{i,5+I}Z_c^{j,5+I}\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{eoc}^{klij}=&\sum_{I=1}^{3}\Big[\Big(\frac{g^2}{8}+\frac{1}{2}l_I^2\Big)\Big(Z_{even}^{k,2+I}Z_{odd}^{l,1}+Z_{even}^{k,1}Z_{odd}^{l,2+I}\Big)\Big(Z_c^{i,1}Z_c^{j,2+I}-Z_c^{i,2+I}Z_c^{j,1}\Big)\\
&-\frac{g^2}{8}\Big(Z_{even}^{k,2}Z_{odd}^{l,2+I}+Z_{even}^{k,2+I}Z_{odd}^{l,2}\Big)\Big(Z_c^{i,2+I}Z_c^{j,2}-Z_c^{i,2}Z_c^{j,2+I}\Big)\Big]\\
&-\frac{g^2}{8}\Big(Z_{even}^{k,1}Z_{odd}^{l,2}+Z_{even}^{k,2}Z_{odd}^{l,1}\Big)\Big(Z_c^{i,1}Z_c^{j,2}-Z_c^{i,2}Z_c^{j,2}\Big)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{cc}^{ijkl}=&\frac{g^2+g'^2}{8}\Big[\sum_{m,n=1}^{5}Z_c^{i,m}Z_c^{j,m}Z_c^{k,n}Z_c^{l,n}+2\sum_{I=1}^{3}Z_c^{i,2+I}Z_c^{j,2+I}\Big(Z_c^{k,1}Z_c^{l,1}-Z_c^{k,2}Z_c^{l,2}\Big)\\
&-2Z_c^{i,1}Z_c^{j,1}Z_c^{k,2}Z_c^{l,2}\Big]+\frac{g'^2}{2}\Big[\sum_{I=1}^{3}Z_c^{i,5+I}Z_c^{j,5+I}\Big(-Z_c^{k,5+I}Z_c^{l,5+I}-Z_c^{k,2+I}Z_c^{l,2+I}\Big)\\
&-\sum_{I=1}^{3}Z_c^{i,5+I}Z_c^{j,5+I}Z_c^{k,1}Z_c^{l,1}\Big]+\sum_{I=1}^{3}l_I^2Z_c^{i,5+I}Z_c^{j,5+I}Z_c^{k,1}Z_c^{l,1}
\end{aligned}
$$

where the mixing matrices Z_{even}, Z_{odd} and Z_c are defined as in Eq. (18), Eq. (22) and Eq. (30).

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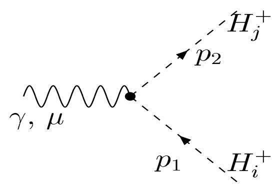
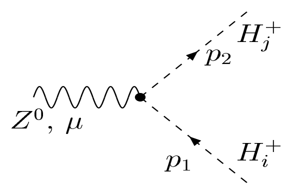
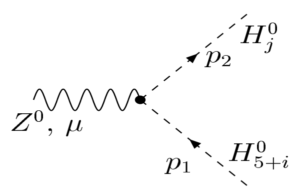
FIGURES



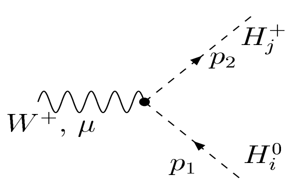
$$-\frac{i}{2}\sqrt{g^2+g\prime^2}\,\sin 2\theta_W\delta_{ij}(p_1+p_2)^\mu$$



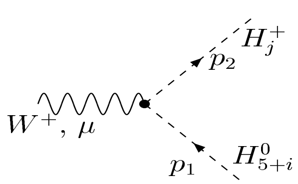
$$\frac{i}{2}\sqrt{g^2+g\prime^2}\Big(\cos 2\theta_W\delta_{ij}-C_c^{ij}\Big)(p_1+p_2)^\mu$$



$$\frac{1}{2}\sqrt{g^2+g\prime^2}C_{eo}^{ij}(p_1+p_2)^\mu$$

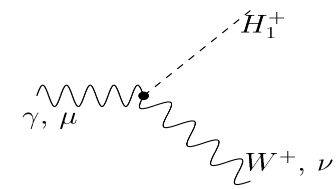


$$\frac{i}{2}\sqrt{g^2+g\prime^2}\cos\theta_W C_{ec}^{ij}(p_1+p_2)^\mu$$

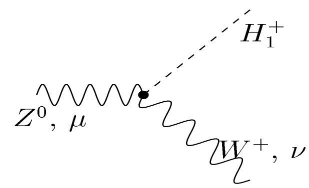


$$\frac{1}{2}\sqrt{g^2+g\prime^2}\cos\theta_W C_{co}^{ij}(p_1+p_2)^\mu$$

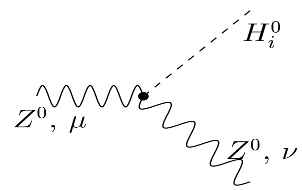
FIG. 1. Feynman rules for SSV vertices, the direction of momentum is indicated above



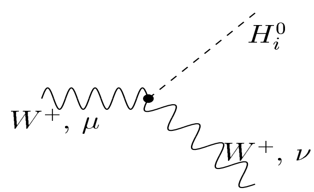
$$-\frac{i}{2}\sqrt{g^2+g\prime^2}\mathrm{M}_W \sin 2\theta_W g_{\mu\nu}$$



$$\frac{i}{2}\sqrt{g^2+g\prime^2}\mathrm{M}_W(\cos 2\theta_W - 1)g_{\mu\nu}$$

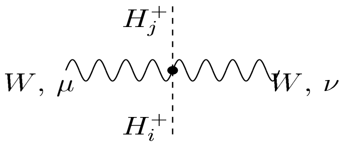


$$-\frac{i}{2}(g^2+g\prime^2)C^i_{even}g_{\mu\nu}$$

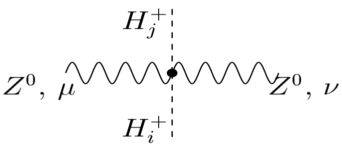


$$-\frac{i}{2}(g^2+g\prime^2)\cos^2\theta_W C^i_{even}g_{\mu\nu}$$

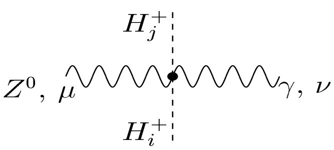
FIG. 2. Feynman rules for SVV vertices



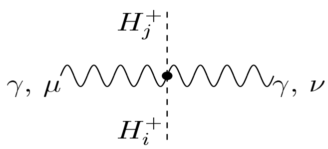
$$-\frac{i}{2}(g^2+g'^2)\cos^2\theta_W\Big(\delta_{ij}-C_c^{ij}\Big)g_{\mu\nu}$$



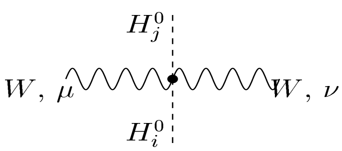
$$-\frac{i}{2}(g^2+g'^2)\Big(\cos^2 2\theta_W\delta_{ij}-C_c^{ij}\Big(4\sin^3\theta_W-\cos^2 2\theta_W\Big)\Big)g_{\mu\nu}$$



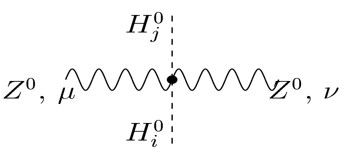
$$-\frac{i}{4}(g^2+g'^2)\Big[\sin 4\theta_W\delta_{ij}-C_c^{ij}\Big(\sin 4\theta_W+8\sin^2\theta_W\cos\theta_W\Big)\Big]g_{\mu\nu}$$



$$-\frac{i}{2}(g^2+g'^2)\sin^2 2\theta_W\delta_{ij}g_{\mu\nu}$$

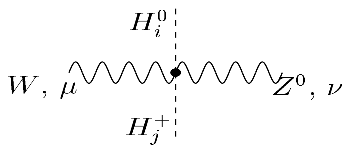


$$-\frac{i}{2}(g^2+g'^2)\cos^2\theta_W\delta_{ij}g_{\mu\nu}$$

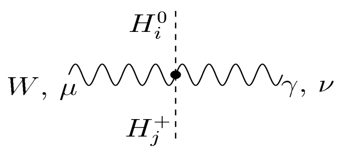


$$-\frac{i}{2}(g^2+g'^2)\delta_{ij}g_{\mu\nu}$$

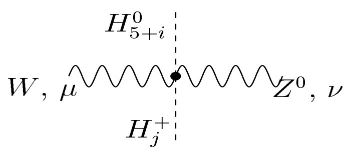
FIG. 3. Feynman rules for SSVV vertices. Part(I)



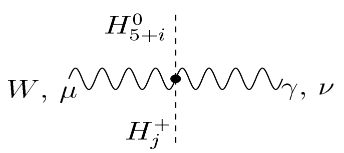
$$\frac{i}{2}(g^2 + g'^2)\cos\theta_W \sin^2\theta_W C_{ec}^{ij} g_{\mu\nu}$$



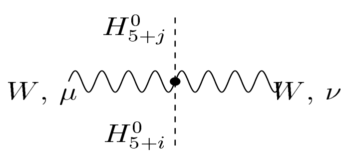
$$-\frac{i}{2}(g^2 + g'^2)\cos^2\theta_W \sin\theta_W C_{ec}^{ij} g_{\mu\nu}$$



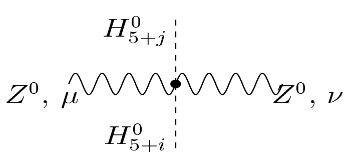
$$-\frac{1}{2}(g^2 + g'^2)\cos\theta_W \sin^2\theta_W C_{co}^{ij} g_{\mu\nu}$$



$$\frac{1}{2}(g^2 + g'^2)\cos^2\theta_W \sin\theta_W C_{co}^{ij} g_{\mu\nu}$$

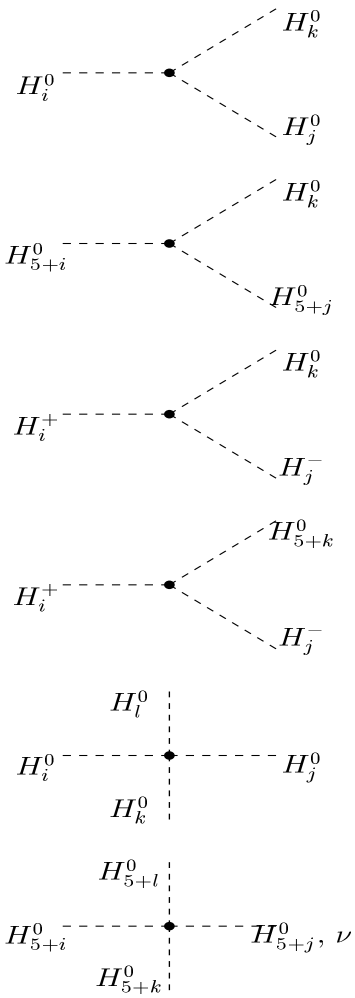


$$-\frac{i}{2}(g^2 + g'^2)\cos^2\theta_W \delta_{ij} g_{\mu\nu}$$



$$-\frac{i}{2}(g^2 + g'^2)\delta_{ij} g_{\mu\nu}$$

FIG. 4. Feynman rules for SSVV vertices. Part(II)



$$-\frac{i}{8}(g^2 + g\prime^2)\mathcal{A}^{ij}_{even}\mathcal{B}^{k}_{even}$$

$$-\frac{i}{8}(g^2 + g\prime^2)\mathcal{A}^{ij}_{odd}\mathcal{B}^{k}_{even}$$

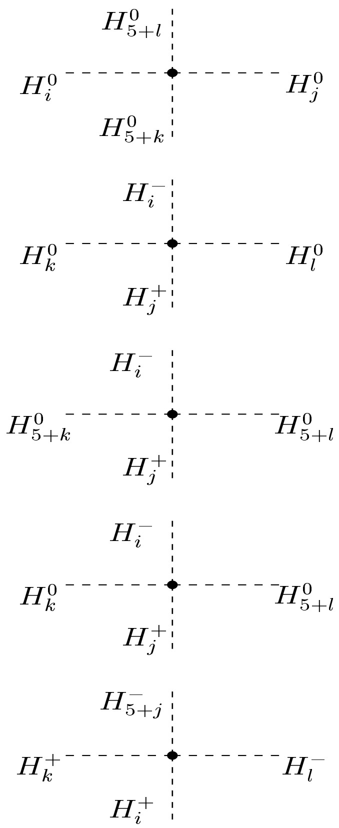
$$-i\mathcal{A}^{kij}_{ec}$$

$$-\mathcal{A}^{kij}_{oc}$$

$$-\frac{i}{32}(g^2 + g\prime^2)\mathcal{A}^{ij}_{even}\mathcal{A}^{ij}_{even}$$

$$-\frac{i}{32}(g^2 + g\prime^2)\mathcal{A}^{ij}_{odd}\mathcal{A}^{ij}_{odd}$$

FIG. 5. Feynman rules for the self-coupling of Higgs. Part(I)



$$\frac{i}{16}(g^2 + g\prime^2)\mathcal{A}^{ij}_{even}\mathcal{A}^{kl}_{odd}$$

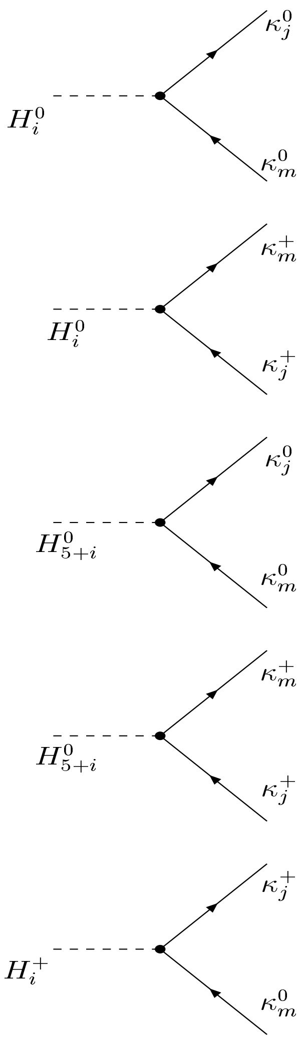
$$-i\mathcal{A}^{klij}_{ec}$$

$$-i\mathcal{A}^{klij}_{oc}$$

$$\mathcal{A}^{klij}_{eoc}$$

$$-i\mathcal{A}^{klij}_{cc}$$

FIG. 6. Feynman rules for the self-coupling of Higgs. Part(II)



$$i\frac{\sqrt{g^2+g\prime^2}}{2}\Big(C_{snn}^{jm}P_L + C_{snn}^{jm*}P_R\Big)$$

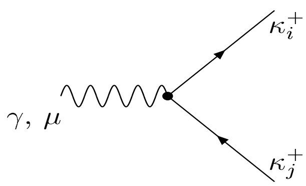
$$i\frac{g}{\sqrt{2}}\Big(C_{skk}^{jm}P_L + C_{skk}^{jm*}P_R\Big)$$

$$-\frac{\sqrt{g^2+g\prime^2}}{2}\Big(C_{onn}^{jm}P_L - C_{onn}^{jm*}P_R\Big)$$

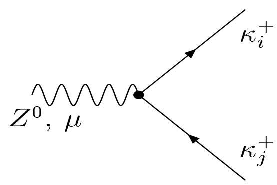
$$-\frac{g}{\sqrt{2}}\Big(C_{okk}^{jm}P_L - C_{okk}^{jm*}P_R\Big)$$

$$i\sqrt{g^2+g\prime^2}\Big(C_{Lnk}^{ij}P_L - C_{Rnk}^{ij}P_R\Big)$$

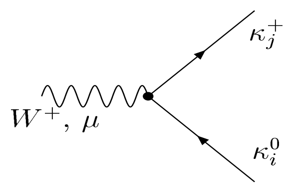
FIG. 7. Feynman rules for the coupling of Higgs with charginos or neutralinos.



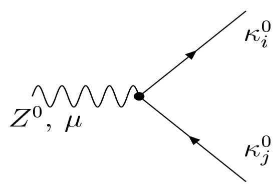
$$i\sqrt{g^2+g\prime^2}\cos\theta_W\sin\theta_W\gamma^\mu\delta_{ij}$$



$$i\sqrt{g^2+g\prime^2}\gamma^\mu[\cos^2\theta_W\delta_{ij} + (\tfrac{1}{2}Z_+^{*i,2}Z_+^{j,2} - \sum_{I=1}^{3} Z_+^{*i,2+I}Z_+^{j,2+I})P_L$$
$$+\tfrac{1}{2}(Z_-^{*i,2}Z_-^{j,2} + \sum_{I=1}^{3} Z_-^{*i,2+I}Z_-^{j,2+I})P_R]$$

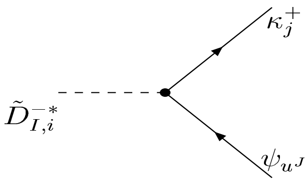


$$ig\gamma^\mu[(-Z_+^{*i,1}Z_N^{j,2} + \tfrac{1}{\sqrt{2}}Z_+^{*i,2}Z_N^{j,4})P_L + (Z_N^{*i,2}Z_-^{j,1}$$
$$+\tfrac{1}{\sqrt{2}}(Z_N^{*i,3}Z_-^{j,2} + \sum_{I=1}^{3} Z_N^{*i,4+I}Z_-^{j,2+I}))P_R]$$

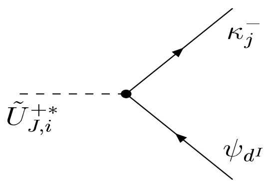


$$i\frac{\sqrt{g^2+g\prime^2}}{2}\Big[\tfrac{1}{2}\Big(Z_N^{*i,4}Z_N^{j,4} - \big(Z_N^{*i,3}Z_N^{j,3} + \textstyle\sum_{\alpha=5}^{7} Z_N^{*i,\alpha}Z_N^{j,\alpha}\big)\Big)P_L$$
$$-\tfrac{1}{2}\Big(Z_N^{*i,4}Z_N^{j,4} - \big(Z_N^{*i,3}Z_N^{j,3} + \textstyle\sum_{\alpha=5}^{7} Z_N^{*i,\alpha}Z_N^{j,\alpha}\big)\Big)P_R\Big]$$

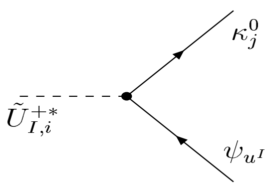
FIG. 8. Feynman rules for the coupling of gauge bosons with charginos or neutralinos.



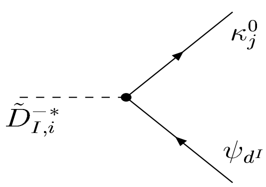
$$iC^{IJ}[(-gZ^{i,1}_{D^I}Z^{j,1}_{-} + \frac{d^I}{2}Z^{i,2}_{D^I}Z^{j,2}_{-})P_L + \frac{u^J}{2}Z^{*j,2}_{+}Z^{i,1}_{D^I}P_R]$$



$$iC^{IJ*}[(-gZ^{i,1}_{U^J}Z^{j,1}_{+} + \frac{u^J}{2}Z^{i,2}_{U^J}Z^{j,2}_{+})P_L - \frac{d^I}{2}Z^{*j,2}_{-}Z^{*i,1}_{U^J}P_R]$$



$$i\{[\frac{\sqrt{g^2+g\prime^2}}{\sqrt{2}}Z^{i,1*}_{U^I}(\cos\theta_W Z^{i,2}_{N} + \frac{1}{3}\sin\theta_W Z^{j,1}_{N}) - \frac{u^I}{2}Z^{i,1*}_{U^I}Z^{j,4}_{N}]P_L$$
$$+[\frac{2\sqrt{2}}{3}g\prime Z^{i,2*}_{U^I}Z^{j,1}_{N} - \frac{u^I}{2}Z^{i,1*}_{U^I}Z^{j,4*}_{N}]P_R\}$$



$$i\{[\frac{\sqrt{g^2+g\prime^2}}{\sqrt{2}}Z^{i,1*}_{D^I}(-\cos\theta_W Z^{i,2}_{N} + \frac{1}{3}\sin\theta_W Z^{j,1}_{N}) + \frac{d^I}{2}Z^{i,1*}_{D^I}Z^{j,3}_{N}]P_L$$
$$+[-\frac{\sqrt{2}}{3}g\prime Z^{i,2*}_{D^I}Z^{j,1}_{N} + \frac{d^I}{2}Z^{i,1*}_{D^I}Z^{j,3*}_{N}]P_R\}$$

FIG. 9. Feynman rules for the coupling of quarks, squarks with charginos or neutralinos.

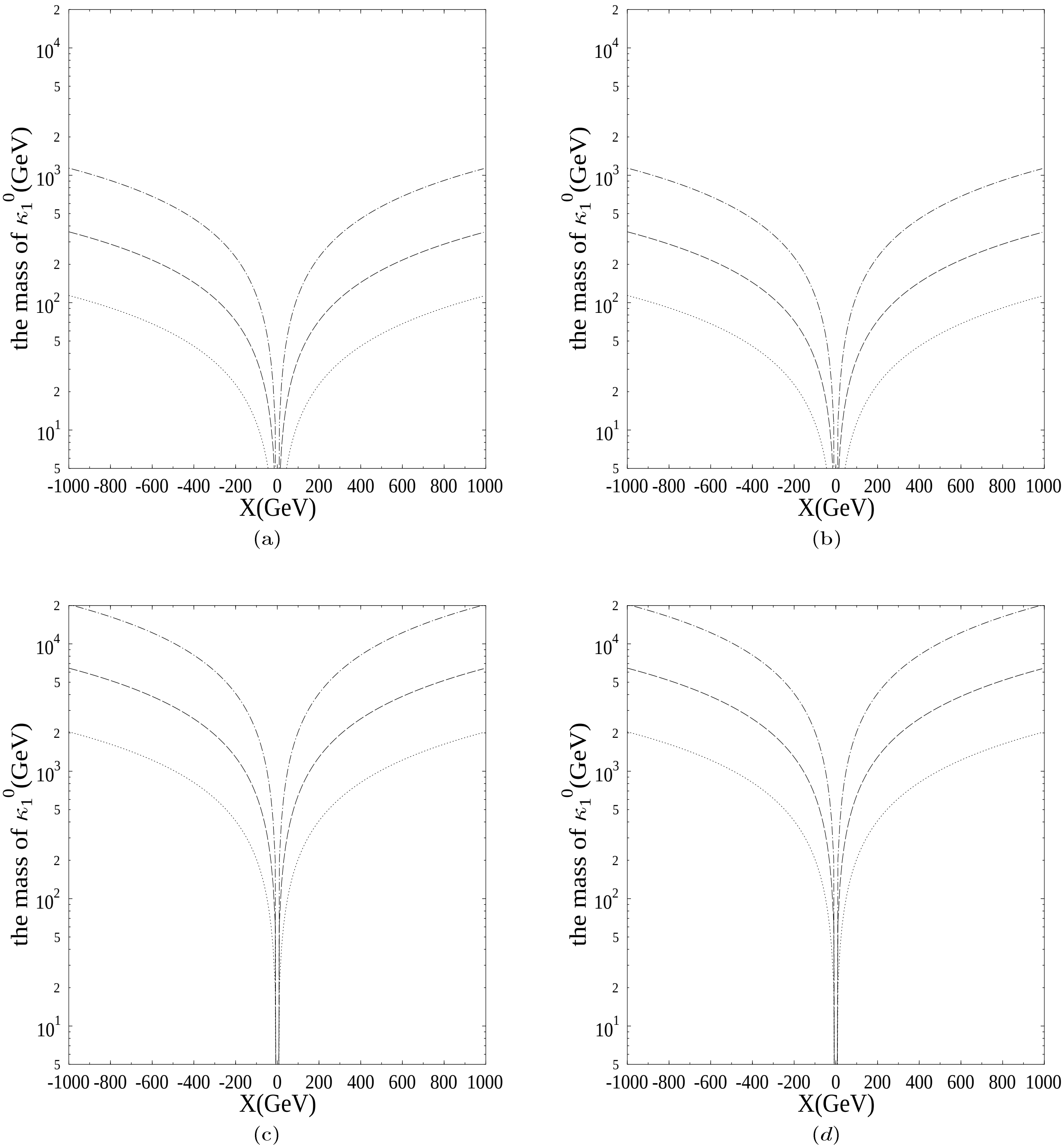


FIG. 10. The mass of the lightest neutralino varied with X. The parameters are assigned as $m_1 = m_2 = 3000$GeV and (a)$\tan\beta = 20$, $\tan\theta_\upsilon = 20$; (b)$\tan\beta = 20$, $\tan\theta_\upsilon = 0.5$; (c)$\tan\beta = 0.5$, $\tan\theta_\upsilon = 20$; (d)$\tan\beta = 0.5$, $\tan\theta_\upsilon = 0.5$. The dot-dash lines correspond to $m_{\nu_\tau} = 0.2$MeV, the dash lines correspond to $m_{\nu_\tau} = 2$MeV, and dot lines correspond to $m_{\nu_\tau} = 20$MeV

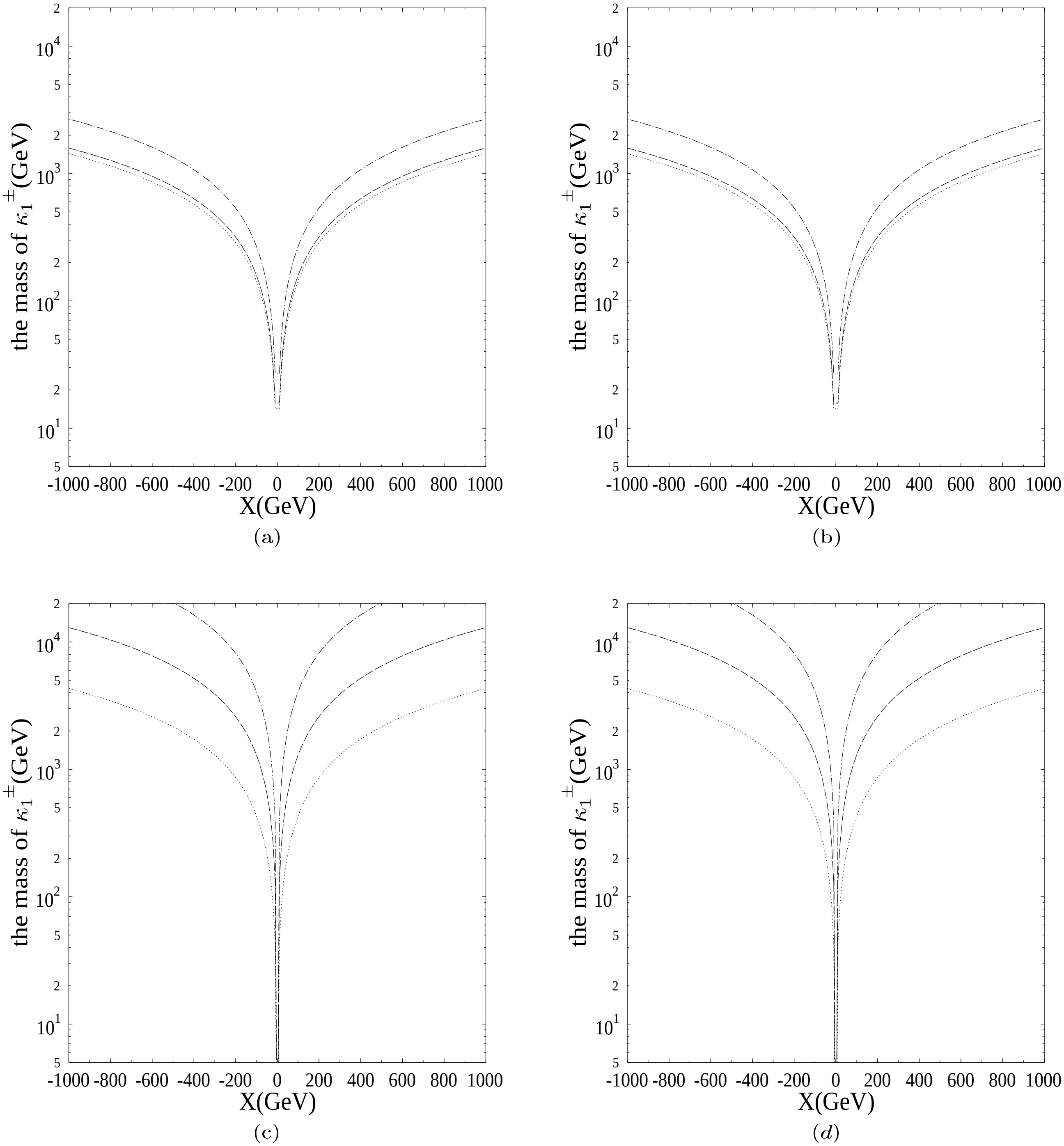


FIG. 11. The mass of the lightest chargino varied with X. The parameters are assigned as $m_1 = m_2 = 3000$GeV and (a)$\tan\beta = 20$, $\tan\theta_\upsilon = 20$; (b)$\tan\beta = 20$, $\tan\theta_\upsilon = 0.5$; (c)$\tan\beta = 0.5$, $\tan\theta_\upsilon = 20$; (d)$\tan\beta = 0.5$, $\tan\theta_\upsilon = 0.5$. The dot-dash lines correspond to $m_{\nu_\tau} = 0.2$MeV, the dash lines correspond to $m_{\nu_\tau} = 2$MeV, and dot lines correspond to $m_{\nu_\tau} = 20$MeV

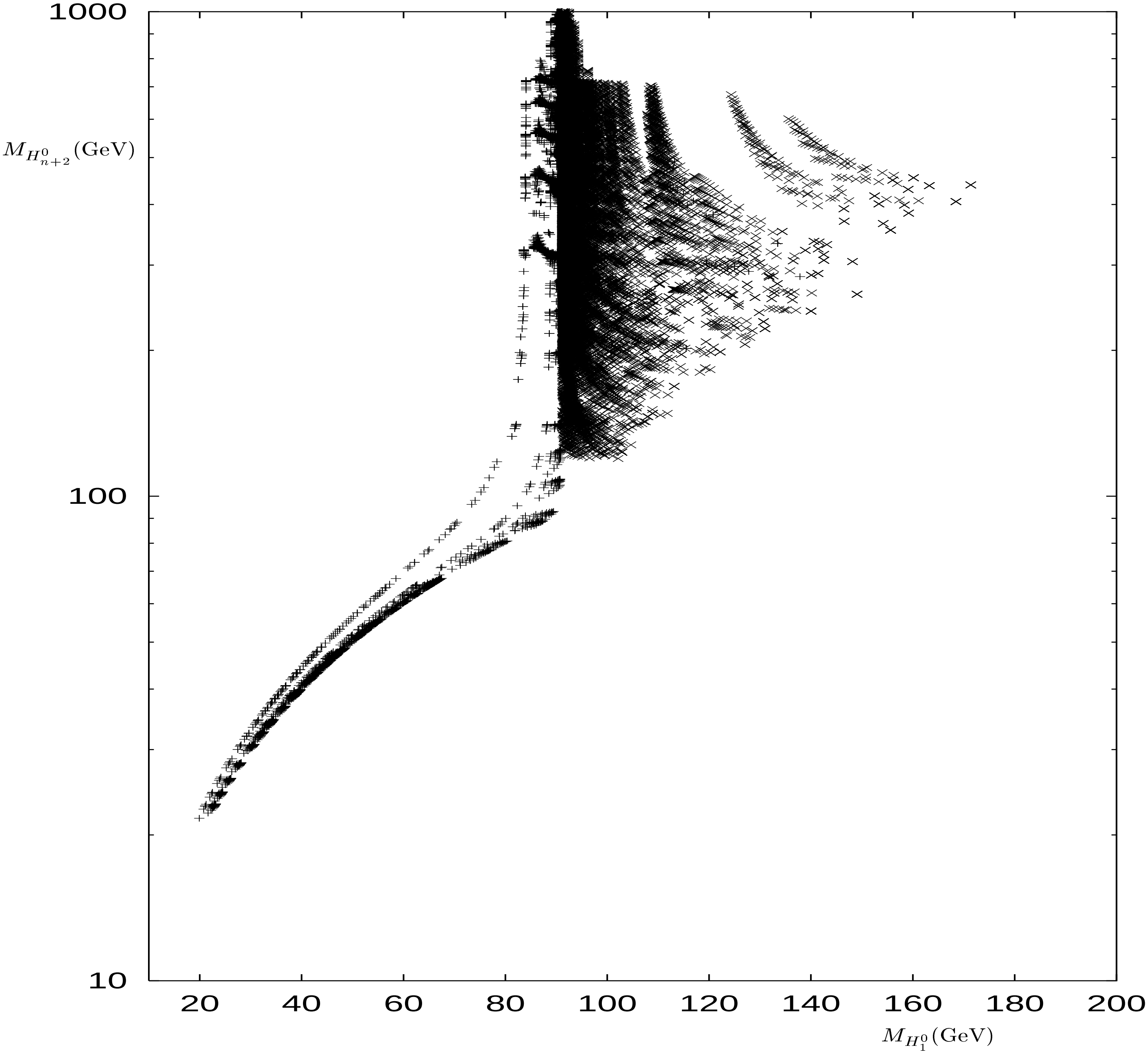


FIG. 12. The mass of the lightest CP-odd Higgs varied with the mass of the lightest CP-even Higgs($n = 3$). The range of parameters are assigned as -10^5GeV^2 $\leq X_s$, Y_s, $Z_s \leq 10^5$ GeV^2 and $0.5 \leq \tan\beta \leq 50$, $0.5 \leq \tan\theta_v \leq 50$.